New look at some parts of free-choice Petri net theory

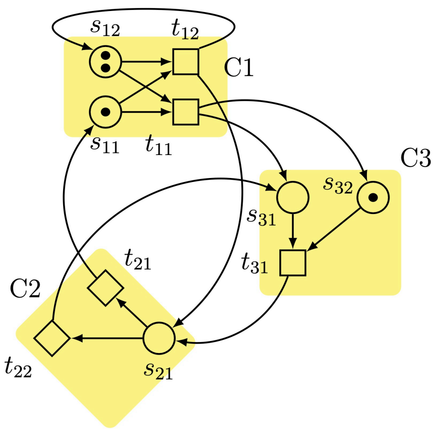
Mgr. Matěj Ošťádal

Supervisor: prof. RNDr. Petr Jančar, CSc. **Palacký University Olomouc | Department of Computer Science**

Palacký University Olomouc

What are Petri nets?

Petri nets are a widely used and established model for analyzing distributed and concurrent systems, in which resources can be absorbed and produced.
A Petri net *N* is a bipartite directed graph with two types of nodes: places (*S*), denoted by circles, and transitions (*T*), denoted by squares.
The edges of the graph are given by the flow relation $F \subseteq (S \times T) \cup (T \times S)$.



A *marking M* assigns tokens to the places of a Petri net.
A transition *t* is enabled if all its input places have a token.
When *t* fires, it takes one token from each input place and adds one to each output place.

Why free-choice?

General Petri nets hold a strong expressive power and the algorithmic analysis of some of their basic behavioural properties is difficult [4]. Our work focuses on free-choice Petri nets — a subclass that strikes a balance between expressive power and practical analysis [2].

Two main behavioural properties of *marked* Petri nets are *liveness* (every transition can eventually be fired) and *boundedness* (the token count does not grow without limit for any of the places).
A Petri net is *well-formed* when there is an *initial* marking for which the net is live and bounded.

For free-choice Petri nets, well-formedness can be checked efficiently — in polynomial time.
The algorithms rely on the Rank Theorem, a deep result from Petri net theory.

Our contribution

A new structural characterization of well-formed free-choice Petri nets was presented in [1]. Our work provides a new proof of this characterization. Our proof omits the use of the *Duality Theorem* used in [1], and relies on the dual notions of *semi-S-components* (*semi-T-components*), which are a relaxed notion of special subnets called *S-components* (*T-components*).

Consequences and further work

The new approach allows us to derive the *Duality Theorem* as a consequence using the notion of *reverse-dual nets*.

Furthemore, the important *Coverability Theorems* are obtained as consequences using the new notions introduced in [1].

We present a new polynomial algorithm for deciding well-formedness, which relies on the previously mentioned well-formedness characterization.
The derived upper bound of $O(|S| \cdot |T| \cdot |F|)$ on the time complexity of our new algorithm is higher than that of the most efficient known algorithms.
On the other hand, our algorithm is significantly easier to understand as it does not rely on nontrivial results.

References

[1] Best, E., Devillers, R., Jančar, P. (2025). Coverability in Well-Formed Free-Choice Nets. In E. Amparore & Ł. Mikulski (Eds.), Application and Theory of Petri Nets and Concurrency (pp. 86–108). Lecture Notes in Computer Science (Vol. 15714). Springer. https://doi.org/10.1007/ 978-3-031-94634-9_5

[2] Desel, J., & Esparza, J. (1995). Free Choice Petri Nets. Cambridge University Press. https://doi.org/10.1017/cbo9780511526558

[3] Kemper, P. (1994). O(|P||T|)-Algorithm to Compute a Cover of S-components in EFC-nets. Forschungsbericht Nr. 543, Department of Computer Science, University of Dortmund.

[4] Jančar, P., & Purser D. (2019). Structural liveness of Petri nets is EXPSPACE-hard and decidable. Acta Informatica, 56(6), 537–552. https://doi.org/10.1007/ s00236-019-00338-6