Texture-Based Analysis of 3D Images

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Motivation

Texture-based image analysis is a field with **numerous applications**, e.g. in the **analysis and classification of 3D medical images**, where it may serve as a fast and reliable tool while **diagnosing illnesses** that cause changes in tissue texture, such as the **Alzheimer's disease** (AD) or various types of **cancer**. Texture analysis specifically focused on 3D images is, thanks to its numerous applications, now a state-of-the-art and rapidly developing discipline.

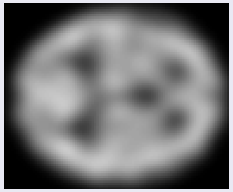

Figure 1: Central cut through a SPECT brain image of a cognitively normal (CN) patient.

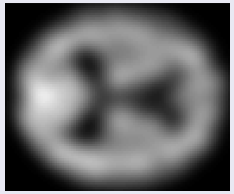

Figure 2: Central cut through a SPECT brain image of a patient with Alzheimer's disease (AD).

In the figures above, we may easily see the **significant differences in texture** between a healthy brain of a cognitively normal (CN) patient and a brain of a patient suffering from Alzheimer's disease with **dead neurons replaced by enlarged fluid-filled ventricles**. Using texture-based analysis, we hope to develop an approach that facilitates fast and correct diagnosis..

Thesis Aims

- Describe several contemporary methods of texture-based image analysis and image classification. Discuss the method of fast, global image filtering with Fourier transform (FT).
- Develop a **new, fast, and reliable algorithm** tailored specifically to the texture analysis of 3D images. This approach will use the **Fourier transform of Zernike polynomials**.
- Implement the proposed algorithm and test it on **single-photon emission computed tomography (SPECT)** images of patients with **Alzheimer's disease**. Calculated image characteristics will serve as inputs of **binary classifiers**. Patients will be classified as either cognitively normal or suffering from Alzheimer's disease.
- **Evaluate the accuracy (ACC)** of classification and suitability of the method for usage in medical diagnostics. Propose several ideas for future development of the method.

Proposed Method

- Using the **convolutional theorem**, it is possible to filter the Fourier transform $\mathrm{F}(\cdot)$ of an n-dimensional image $\mathrm{f}(\cdot)$ in the frequency domain by using the element-wise multiplication with a chosen kernel $\mathrm{K}(\cdot)$: $\mathrm{G}(\cdot) = \mathrm{K}(\cdot) \otimes \mathrm{F}(\cdot)$.
- Let $\boldsymbol{r} = (r, \theta, \phi)$ be a vector of three-dimensional spherical coordinates, $r \in [0, 1]$, $\theta \in [0, \pi]$, $\phi \in (-\pi, \pi]$. Let $\mathcal{B}$ be a **unit ball** in L^2. Let $\mathcal{P} = [0, 1] \times [0, \pi] \times (-\pi, \pi]$ be the representation of $\mathcal{B}$ in spherical coordinates. Let $\mathcal{B}_0$ be a **unit sphere** in L^2. Let $\mathcal{P}_0 = [0, \pi] \times (-\pi, \pi]$ be the representation of $\mathcal{B}_0$ in spherical coordinates.
- Let $l, n \in \mathbb{N}_0$ and $m \in \mathbb{Z}$, $-l \leq m \leq l$. **Zernike polynomial** $Z^m_{l,n}\colon \mathcal{P} \to \mathbb{C}$ is defined as $Z^m_{l,n}(r, \theta, \phi) = R_{l,n}(r) Y^m_l(\theta, \phi)$, where $R_{l,n}\colon [0, 1] \to \mathbb{R}$ is a **radial polynomial** and $Y^m_l\colon \mathcal{P}_0 \to \mathbb{C}$ a **spherical harmonic function** [1].
- To obtain the **FT of a Zernike polynomial** $Z^m_{l,n}(\boldsymbol{r})$ we use the equation [1]:

$$\mathscr{F}(Z^m_{l,n}(\boldsymbol{r})) = \frac{(-1)^n}{\mathrm{i}^l 2\pi^{l+\frac{1}{2}}} Y^m_l\left(\frac{\boldsymbol{k}}{k}\right) \frac{J_{2n+l+\frac{3}{2}}(k)}{k}, \tag{1}$$

where $\boldsymbol{k} = (k, \psi, \mu)$ is the coordinate vector of a point in the three-dimensional frequency domain, and $J_\alpha\colon \mathbb{R} \to \mathbb{R}$ is a Bessel function of the first kind and order $\alpha \in \mathbb{R}^+_0$.

Proposed Method (cont.)

- Let $f\colon \mathcal{P} \to \mathbb{C}$ be a general complex-valued function. The **linear expansion** of the function $f(\boldsymbol{r}) \in \mathcal{B}$ is defined by the formula [1][2]:

$$f(\boldsymbol{r}) = \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m=-l}^{l} a_{l,n,m} Z^m_{l,n}(\boldsymbol{r}), \tag{2}$$

The linear combination (2) is **SO**(3) **rotationally invariant** [1]. The coefficients $a_{l,n,m}$ are given by the formula stemming from the **orthonormality of Zernike polynomials** [2]:

$$a_{l,n,m} = \int_0^1 \int_0^{\pi} \int_{-\pi}^{\pi} f(\boldsymbol{r}) \overline{Z^m_{l,n}(\boldsymbol{r})} \mathrm{d}\boldsymbol{r}, \tag{3}$$

where $\mathrm{d}\boldsymbol{r} = r^2 \sin(\theta) \mathrm{d}r \mathrm{d}\theta \mathrm{d}\phi$.

- Let $\mathrm{f}\colon \mathcal{S} \to \mathbb{C}$ be a complex-valued three-dimensional image with a support set $\mathcal{S}$. Let $\mathrm{F}(\cdot)$ be its frequency domain representation. Let $\mathrm{Q}^m_{l,n}(\cdot)$ be the frequency domain representation of $\overline{Z^m_{l,n}(\boldsymbol{r})}$. Let $\mathrm{A}_{l,n,m}\colon \mathcal{S} \to \mathbb{C}$ be a three-dimensional image, whose element intensities correspond to spatial representation of coefficients $a_{n,l,m}$. By using the convolutional theorem, we may see that: $\mathrm{A}_{l,n,m}(\cdot) = \mathscr{F}^{-1}(\mathrm{Q}^m_{l,n}(\cdot) \otimes \mathrm{F}(\cdot))$.
- We define the l, n-th invariant $b_{l,n}$ as a sum $b_{l,n} = \sum_{m=-l}^{l} |a_{l,n,m}|^2$ and observe that $\mathrm{B}_{l,n}\colon \mathcal{S} \to \mathbb{R}$ consisting of values $b_{l,n}$ is a three-dimensional real-valued image, whose **intensities correspond to a discrete sampling of the rotational invariant**.

Implementation and Data

- Implementation of the proposed algorithm in the **MATLAB** environment. Combination of own implementation and existing toolboxes.
- 55 AD and 56 CN SPECT $^{99\mathrm{m}}$Tc-HMPAO images of $79 \times 95 \times 69$ voxels of size $1 \times 1 \times 1$ mm from the Nuclear Medicine Center of Fakultní nemocnice Královské Vinohrady in Prague [3].
- Optional use of **low-pass (LP) and high-pass (HP) global filtering** (smoothing and sharpening) as data preprocessing.
- After calculating the invariants, we again easily see the **difference between CN and AD brain** (note the asymmetry in fig. 4 and compare to fig. 2):

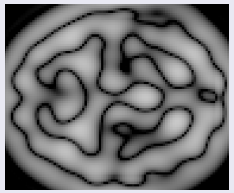

Figure 3: Central cut through the invariant $b_{1,0}$ of a brain of a CN patient.

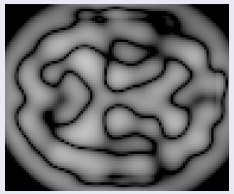

Figure 4: Central cut through the invariant $b_{1,0}$ of a brain of a patient with AD.

- **Global texture characteristics** of rotational invariants such as the minimum, maximum, range, mean, median and other percentiles, median absolute deviation (MAD), variance, skewness, and kurtosis.
- Depending on the number of used invariants, we may obtain hundreds or thousands of image characteristics. We use three approaches to **preprocess large data** to serve as inputs of binary classifiers:
  - Full data - all calculated characteristics together as one large input.
  - One-dimensional data - all calculated characteristics one by one.
  - Reduced data - data dimension reduced to ≤ 10 using the **principal component analysis (PCA)**.
- **Binary classification** using tools such as the linear discriminant analysis (LDA), quadratic discriminant analysis (QDA), method of k-nearest neighbours (KNN), support vector machines (SVMs), and artificial neural networks (ANNs).

Results

- Nine invariants for $l, n \leq 2$, twelve characteristics and one configuration with no filter preprocessing as well as one configuration with Butterworth low-pass filter preprocessing. Number of characteristics $p = 9 \cdot 12 \cdot 2 = 216$. **Leave-one-out** cross-validation method to test the trained classifiers.
- Only LDA, QDA and KNN used for full data, as SVMs and ANNs would be too slow due to a large number of inputs. Best results in true positive rate (TPR), true negative rate (TNR) and accuracy (ACC) achieved with QDA with regularization parameter $\lambda = 0.05$. KNN with $k = 11$ achieved significantly worse results than both LDA and QDA.

Table 1: Comparison of best classifier configurations for full data.

	LDA, $\lambda = 0.03$	QDA, $\lambda = 0.05$	KNN, $k = 11$
TPR	0.820	0.900	0.700
TNR	0.880	0.880	0.800
ACC	0.850	0.890	0.750

- Good performance of some of the single characteristics while testing one-dimensional data. $\mathrm{ACC} = 0.770$ and $\mathrm{ACC} = 0.740$ achieved with **kurtosis** and **skewness** of the invariant $b_{0,0}$. Median of the invariant $b_{2,0}$ and skewness of the invariant $b_{2,2}$ both resulted in $\mathrm{ACC} = 0.700$.
- Best classification results achieved with PCA reduced data serving as inputs of SVMs and ANNs. Best results overall, $\mathrm{ACC} = 0.920$, achieved with data reduced to seven dimensions and a **Gaussian SVM** with scaling parameter set to $\sigma = 1$.

Table 2: Comparison of the best classifier configurations for reduced data.

	SVM, Gauss., $\sigma = 1$, PCA $= 7$	ANN, Sigm., Neur. $= 5$, PCA $= 3$
TPR	0.940	0.920
TNR	0.900	0.880
ACC	0.920	0.900

Conclusions

- Successful development, implementation, and testing of a **new and fast algorithm** to compute **rotationally invariant global texture characteristics of 3D images**.
- Method used for **classification of SPECT images** of the brains of patients suffering from Alzheimer's disease.
- **Fast performance** and **high accuracy** ($\mathrm{ACC} = 0.920$) in patient classification. Method **suitable for further development** and testing in medical diagnostics on more varied types of medical data (larger AD/CN dataset, patients with cancer, etc.).
- Good classification results achieved while using some of the one-dimensional characteristics suggests that it would be reasonable to develop a theoretical approach to enable the algorithm to efficiently **select some of the calculated characteristics** as inputs to binary classifiers, which would **reduce computation costs** and potentially **increase classification accuracy**.

References

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