

# Sparse Approximate Inverse for Enhanced Scalability in Recommender Systems



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## Motivation

Shallow neural networks are simple yet often outperform deep learning approaches in collaborative filtering tasks [1]. **Embarrassingly Shallow Autoencoder (EASE<sup>R</sup>)** [2] is a linear model, which – despite its simplicity –

- aggregates feedback from all users to compensate for scarce feedback from individuals
- uses *long chains* of user–item feedback to model item similarity.

Instead of gradient descent, the training procedure uses *closed-form solution* of its convex optimization objective, improving training complexity. However, this process relies on the calculation of  $A^{-1} = (X^T X + \lambda I)^{-1}$ , introducing **two challenges for practical application**:

1. Computing  $A^{-1}$  is **costly** (but depends only on #items).
2. Despite the sparsity of input data  $X$ ,  $A^{-1}$  (and also the weights) will be **dense**.

Crucially, the model must fit in RAM for inference.  
**1M items** → **model size = 4 TB** (in float32).

## Conclusion

Popular shallow autoencoder EASE<sup>R</sup> leverages long user–item interaction chains. This ability positively affects the quality of recommendations but also prohibitively increases training and inference costs on large item sets. We introduce a solution to these problems using modern numerical methods for sparse approximate inversion. The techniques are scalable and robust enough to find critical (even long-distance) information. By exploiting the inherent sparsity of user–item interaction data, our end-to-end sparse method achieves substantial efficiency gains over previous approaches that attempt to overpower the problem using dense block operations. The resulting model SANSa provides a robust yet attainable baseline model for researchers with limited resources and large-scale industry environments with millions of items.

The thesis outcomes were presented at an international conference on recommender systems [3]. The model is currently under testing for production deployment.

## Highlights of the proposed method

- Alleviate the main drawback of a broadly used EASE<sup>R</sup> recommendation algorithm
- **Cheap & easy-to-use** for researchers & **scalable** enough even for large industrial settings

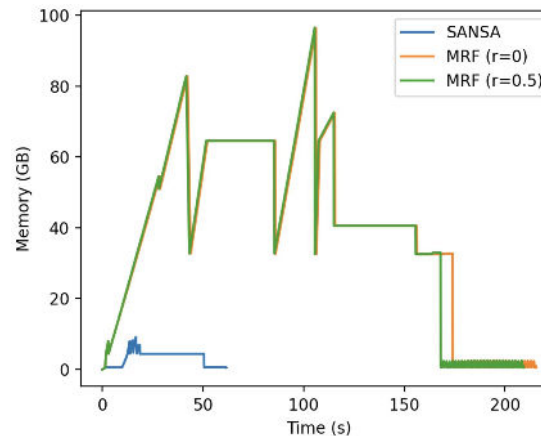
## Experiment results

- demonstrate robustness and efficiency on 5 datasets
- Amazon Books: 53K users, **92K items**, 3M interactions

	Amazon Books						
	SANSa (ICF)	MRF ( $r=0$ )	MRF ( $r=0.5$ )	EASE <sup>R</sup>	SLIM	ITEMCF	ULTRAGCN
recall@20	<b>0.077</b>	0.071	0.069	0.071	0.075	0.074	0.068
nDCG@20	<b>0.064</b>	0.058	0.055	0.057	0.060	0.061	0.056
Training resources							
vCPU	2	16	16	28	28	28	20*
memory usage (GB):							
peak	<b>9.18</b>	96.45	96.58	---	---	---	---
average	<b>3.87</b>	49.12	49.75	---	---	---	---
time	<b>49 s</b>	172 s	167 s	222 m	316 m	57 m	45 m

\*and a GPU (RTX 2080)

- 3x faster training with 10x less memory compared to previous sparse modification of EASE<sup>R</sup> – MRF [4]
- **orders of magnitude faster and cheaper** than other models
- new state-of-the-art accuracy on the dataset



## How to scale EASE<sup>R</sup> to millions of items?

Approximate EASE<sup>R</sup> using a sparse model

- preserve properties of  $A^{-1}$  — full rank, SPD
- enable **arbitrary model compression** — allow users to specify weight density of the resulting model

Method: **factorized sparse approximate inversion**

- sophisticated approaches developed for numerical solvers [5]
- **extract global dominant information** from user–item interaction graph
- $A$  is SPD → increased efficiency, higher compression

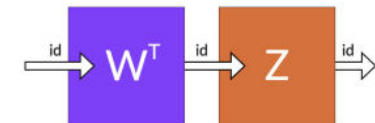
The approximate inverse is computed in 3 steps:

1. approximate (or incomplete) *sparse* Cholesky factorization
2. *free* initial approximation of the inverse factor
3. refinement based on Frobenius norm minimization

## Model – training and architecture

Scalable Approximate NonSymmetric Autoencoder (SANSa)

- 1: **input** user–item interaction matrix  $X$ , L2 regularization  $\lambda$
- 2: compute sparse  $LDL^T \approx P(X^T X + \lambda I)P^T$  (for a permutation  $P$ )
- 3: compute sparse  $K \approx L^{-1}$
- 4:  $W \leftarrow KP$
- 5:  $Z_0 \leftarrow D^{-1}W$
- 6:  $\vec{r} \leftarrow \text{diag}(W^T Z_0)$
- 7:  $Z \leftarrow$  scale the columns of  $Z_0$  by  $-1/\vec{r}$
- 8: **return**  $W^T, Z$



## References

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