

# On Erasing Rules in Regulated Grammars

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## Abstract

**T**HIS work discusses the effect of erasing rules to the generative power of regulated grammars, which is a big open problem in the theory of regulated rewriting. It studies the possibility of removal of erasing rules from regulated grammars by aggregation of current, up-to-date results concerning this elimination and by presentation of a new condition, called  $k$ -limited erasing, under which all erasing rules can be always removed from regularly controlled context-free grammars without affecting their generative power. This result partially solves the abovementioned problem. Moreover, a new algorithm for elimination of erasing rules from context-free grammars is presented. This algorithm does not require any predetermination of so called  $\varepsilon$ -nonterminals (in contrast to the standard algorithm used in textbooks). In the conclusion, a significance of these results concerning syntactical analysis is discussed.

## 1. Introduction

**O**VER its history, the formal language theory has intensively and systematically investigated various grammars with regulated derivations. In many respects, this theory has established their fundamental properties. Nevertheless, this investigation area still represents a vividly discussed area of the language theory as demonstrated by several recent studies. Indeed, there still remain some crucially significant open problems concerning these grammars, including the exact effect of erasing rules to the generative power of regulated grammars.

Whereas the (im)possibility of erasing rules elimination from some types of regulated grammars was proven, there are still some regulated grammars where the possibility of erasing rules removal is still an open problem. More specifically, for example, it is not known whether we are always able to

remove all erasing rules from any regularly controlled context-free grammar, matrix grammar, programmed grammar, random context grammar, ordered grammar, and Russian parallel grammar, without affecting the generated language.

The goals of this work are (1) to present current, up-to-date results concerning the exact effect of erasing rules to the generative power of regulated grammars and the possibility of elimination of such rules from these grammars, (2) to study new results in this area, and (3) to discuss a significance of these results concerning syntactical analysis. Thereby, this work contributes to the theory of regulated rewriting, which is an important field of the formal language theory.

## 2. Results

### Present Results

- Overview of known results regarding the possibility of elimination of erasing rules from various regulated grammars (for example, we are able to eliminate all erasing rules from any indexed grammar and any Indian parallel grammar without affecting the generated language).
- Limited erasing in scattered context grammars: even though it is not possible to remove all erasing rules from any scattered context grammar, we are able to do this in case of scattered context grammars satisfying a certain condition.
- Recursive erasing in programmed grammars: we are able to remove all so-called *recursively erasing rules* from any programmed grammar.
- Generation of extended languages by grammars without erasing rules: we can generate *quotients* and *coincidental extensions* of the original languages to remove erasing rules from some types of regulated grammars.
- Erasing in Petri net languages and matrix grammars: as Petri nets are related to specific models from the formal language theory, one can use particular results from the theory of Petri

nets and apply them in the formal language theory. Some of these results shed a new light on the question whether it is possible to eliminate all erasing rules from any matrix grammar.

### New Results

- An algorithm for elimination of all erasing rules from regularly controlled context-free grammars satisfying a certain condition, called *k-limited erasing*. This algorithm represents a partial solution to the problem concerning the effect of erasing rules to the generative power of these grammars.
- An alternative algorithm for elimination of all erasing rules from context-free grammars, which does not require any predetermination of so called  $\varepsilon$ -nonterminals (in contrast to the standard algorithm used in textbooks).

## 3. Conclusion

**N**OTICE that this partial solution to the problem concerning the effect of erasing rules to the generative power of regularly controlled context-free grammars is of an interest in the formal language theory. Indeed, as erasing rules do not effect the power of regular-controlled context-free grammars satisfying the condition, this theory can narrow its future investigation concerning this problem only to the grammars in which this condition is unsatisfied.

To conclude the significance of elimination of erasing rules from regulated grammars to syntactical analysis, the two main problems are the lack of advanced, thoroughly examined, and generally usable parsing methods based on regulated grammars and high syntactic complexity of resulting grammars without erasing rules. Further research in these two directions is necessary. Until then, the value of the results concerning the elimination of erasing rules from regulated grammars is mainly theoretical.