BALANCED USE OF RESOURCES IN COMPUTATIONS

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We study the concept of balanced use of resources in deterministic sequential computations and continue in the research initiated in [Kováč, 2010] and [Kostolányi, 2011].

We define and analyze several types of balanced use of resources (i.e., *equiloadedness*) for various types of deterministic automata. In order to make our definitions independent of a particular model of computation, we present an abstraction of one-way deterministic automata inspired by abstract families of automata of S. Ginsburg – *abstract deterministic automata* (ADA).

We present several definitions of equiloadedness for abstract deterministic automata which generalize the definitions presented in [Kostolányi, 2011] and [Kováč, 2010]. Informally, an abstract deterministic automaton is said to be *strictly state-$\mathcal{S}$-equiloaded* (*strictly transition-$\mathcal{S}$-equiloaded*), if each its state (transition) is used the same number of times (up to a constant difference) in computation paths from a given set $\mathcal{S}$.

In order to define the remaining types of equiloadedness, we define the concept of the *state-equiloadedness $\mathcal{S}$-measure* and of the *transition-equiloadedness $\mathcal{S}$-measure*. These measures assign to every abstract deterministic automaton a real number from the interval $[0, 1]$, with the value 1 representing the most balanced use of a given resource, and the value 0 representing the least balanced use of a given resource. An ADA is said to be *state-$\mathcal{S}$-equiloaded* (*transition-$\mathcal{S}$-equiloaded*), if its state-equiloadedness (transition-equiloadedness) $\mathcal{S}$-measure is 1. An ADA is said to be *weakly state-$\mathcal{S}$-equiloaded* (*weakly transition-$\mathcal{S}$-equiloaded*), if its state-equiloadedness (transition-equiloadedness) $\mathcal{S}$-measure is greater than 0. Strict $\mathcal{S}$-equiloadedness is proved to be a stronger requirement than $\mathcal{S}$-equiloadedness.

In this work we extend the study of equiloaded DFA studied so far in several ways. We generalize the definitions for DFA to ADA and study also equiloaded DFAε (DFA with ε-transitions) and DOCA (deterministic one-counter automata). Besides, we prove also some results that hold for ADA in general.

We also consider more general sets of computations with the set of accepting computations studied so far being just a particular case.

By proving the equivalence of certain definitions, we unify the theories for DFA from [Kováč, 2010] and from [Kostolányi, 2011]. We also study the family of weakly state-$\mathcal{S}$-equiloaded DFA that has not been studied yet. For this family of automata, we prove a characterization based on Perron–Frobenius eigenvalues of their strongly connected components. Furthermore, we improve our theory that allows us to express the basic quantities related to equiloadedness as solutions to initial value problems for homogeneous systems of first-order linear OΔEs with constant coefficients and to efficiently use the Perron–Frobenius theory to study their asymptotic properties. Finally, we generalize most of the results for DFA also to DFAε (and prove that equiloaded DFAε have greater computational power than equiloaded DFA).

In this work, we also initiate the study of strictly $\mathcal{S}$-equiloaded DOCA. Most importantly, we prove the characterization of strictly transition-$\mathcal{S}$-equiloaded DOCA and prove that this property is decidable.

Finally, we present results concerning relations between families of equiloaded languages and their closure properties.

References

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