Automated Generation of Planar Geometry Olympiad Problems

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Motivation

The **International Mathematical Olympiad** is the most prestigious mathematical contest.

http://imo-official.org/

- 100+ countries
- 600+ participants
- 6 difficult problems
- 2 planar geometry problems
- **Writing these problems** is:
  - Difficult
  - Time-consuming
  - Requires years of experience

Main challenges

- **Very little prior work** in automation
- Multitude of possible problems
- Only a minority of them is suitable
- **No universal way** of recognizing them

Methods

1. **Generation of geometry problems**
   - Complex algorithm with **efficient memory usage**
   - Allowed for arbitrarily long-running experiments
   - **Formally proven** to be correct
2. **Filtering unsuitable problems**
   - Ulitizes **geometry theorem proving** methods
   - Filtered out **95%** of the easy problems in the performed test-case experiment
3. **Ranking of the remaning problems**
   - **Heuristic** ranking system
   - **4 rated aspects** selected based on the author's years of **experience** in writing geometry problems

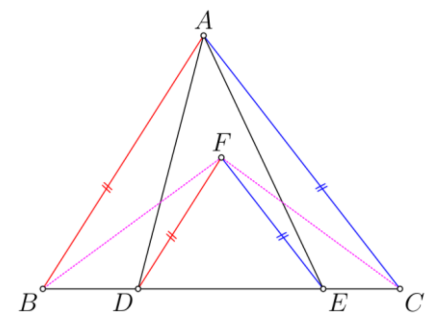
Implementation

- C# .NET Core 3.1
- **No external libraries** for the main logic
- Visualization via MetaPost and TeX
- https://github.com/PatrikBak/GeoGen

Results

- Tested in long-running (40+ hours) parallel generations (45,000+ CPU hours)
- More than **100,000 problems**
- 5 problems proposed to the **International Mathematical Olympiad 2020**
- 1 problem accepted to the **Czech-Slovak-Polish Match 2020**
- 4 problems accepted to the **Czech-Slovak Olympiad 2020**

Accepted problem



Let ABC be an acute triangle. Suppose that points D and E lie on the side BC such that D is between B and E, $AD = CD$, and $AE = BE$. Point F is a point satisfying $FD \parallel AB$ and $FE \parallel AC$. Prove that $FB = FC$.