Satisfiability of DQBF Using Binary Decision Diagrams

Juraj Síč (supervisor: Jan Strejček)

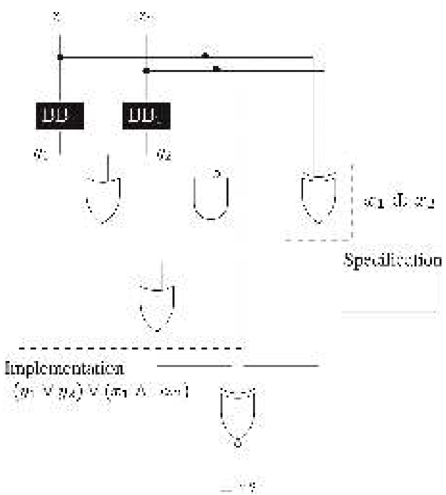
Faculty of Informatics, Masaryk University

Problem

- Decide satisfiability of a given DQBF
- DQBF = **dependency quantified Boolean formula**
  - Propositional logic formula extended with **quantifiers with explicit dependencies** between them
- **NEXPTIME**-complete problem
- Example:

$$\forall x_1 \forall x_2 \exists y_1(x_1) \exists y_2(x_2).(x_1 \wedge x_2) \Leftrightarrow (y_1 \Leftrightarrow y_2)$$

  - y_1 depends only on x_1 (and y_2 only on x_2), meaning that the value of y_1 cannot change based on the value of x_2
  - Formula is **unsatisfiable** as y_1 and y_2 cannot coordinate
- Can be used for solving
  - **controller synthesis problem** (CSP)
  - **partial equivalence checking** (PEC) –Can a combinational circuit with black boxes (BB) be equivalent to a given specification?

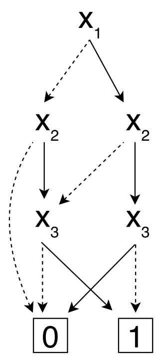


- Figure above [2] shows a PEC problem encoded by a DQBF

$$\forall x_1 \forall x_2 \exists y_1(x_1) \exists(x_2).((y_1 \vee y_2) \vee (x_1 \wedge \neg x_2)) \Leftrightarrow (x_1 \oplus x_2)$$

Method

- **Quantifier elimination** is used as the basic solving technique
  - Quantifiers are iteratively eliminated until we end up with True or False
- Algorithm improved by **quantifier localisation**
  - Quantifiers are pushed inside the formula resulting in a faster elimination
- **Binary decision diagrams** (BDDs) are used to represent propositional subformulas in DQBF
- The BDD on the right represents $(\neg x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge \neg(x_2 \Leftrightarrow x_3))$



Results

Quantifier localisation improvements

- Correction of existing results
- Proved that it can be used in subformulas
- Proved that universal quantifier elimination can be done locally

Solver **DQBDD**

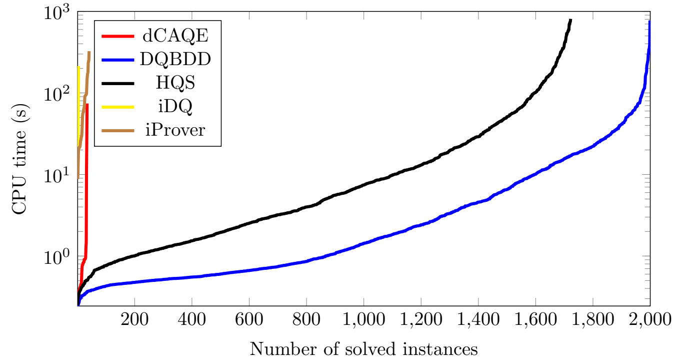
- New algorithm solving DQBF satisfiability
- Implemented in C++ using BDDs
- **Winner** of the DQBF track of **QBFEval'20** competition [1]

Publications under preparation

- Joint journal paper with the HQS team (University of Freiburg)
- Publication about the new algorithm of DQBDD

Experiments

- Comparison of possible quantifier localisation and elimination strategies
- Comparison of DQBF solvers using different benchmarks
- Results:
  - **DQBDD** is far **better** than other solvers for PEC
  - Figure below shows a cactus plot comparing runtimes of DQBF solvers for PEC instances



  - **DQBDD** is nearly **as good as** the best solvers for CSP

QBFEval'20 Competition

- Comparison of DQBF solvers on selected benchmarks
- Results
  1. DQBDD –257 solved in 5396 s
  2. HQS –195 solved in 2662 s
  3. iProver –170 solved in 17399 s

References

[1] QBF evaluation 2020. `http://www.qbflib.org/qbfeval20.php`.

[2] Karina Gitina et al. Equivalence checking of partial designs using dependency quantified boolean formulae. In **ICCD'13**.