Satisfiability of DQBF Using Binary Decision Diagrams

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Problem

- Decide satisfiability of a given DQBF
- DQBF = dependency quantified Boolean formula
  - Propositional logic formula extended with quantifiers with explicit dependencies between them
- NEXPTIME-complete problem
- Example:
  \[ \forall x_1 \forall x_2 \exists y_1 (x_1) \exists y_2 (x_2) \cdot (x_1 \land x_2) \iff (y_1 \leftrightarrow y_2) \]
  - \( y_1 \) depends only on \( x_1 \) (and \( y_2 \) only on \( x_2 \)), meaning that the value of \( y_1 \) cannot change based on the value of \( x_2 \)
  - Formula is unsatisfiable as \( y_1 \) and \( y_2 \) cannot coordinate
- Can be used for solving
  - controller synthesis problem (CSP)
  - partial equivalence checking (PEC) - Can a combinational circuit with black boxes (BB) be equivalent to a given specification?

Method

- Quantifier elimination is used as the basic solving technique
  - Quantifiers are iteratively eliminated until we end up with True or False
- Algorithm improved by quantifier localisation
  - Quantifiers are pushed inside the formula resulting in a faster elimination
- Binary decision diagrams (BDDs) are used to represent propositional subformulas in DQBF
  - The BDD on the right represents \( (\neg x_1 \land x_2 \land x_3) \lor (x_1 \land \neg (x_2 \iff x_3)) \)

Results

- Quantifier localisation improvements
  - Correction of existing results
  - Proved that it can be used in subformulas
  - Proved that universal quantifier elimination can be done locally

Solver DQBDD

- New algorithm solving DQBF satisfiability
- Implemented in C++ using BDDs
- Winner of the DQBF track of QBFEval’20 competition [1]

Experiments

- Comparison of possible quantifier localisation and elimination strategies
- Comparison of DQBF solvers using different benchmarks
- Results:
  - DQBDD is far better than other solvers for PEC
  - Figure below shows a cactus plot comparing runtimes of DQBF solvers for PEC instances

QBFEval’20 Competition

- Comparison of DQBF solvers on selected benchmarks
- Results
  1. DQBDD – 257 solved in 5396 s
  2. HQS – 195 solved in 2662 s
  3. iProver – 170 solved in 17399 s

References