Application of data dependent discrete Laplacian

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Abstract

Discrete Laplace operator has a wide spectrum of applications in mesh processing, for example in smoothing, parameterization, editing and compression. In the latter, Váša et. al. [1] have shown, that using a geometric discrete Laplace operator results in residual entropy reduction, when compressing dynamic meshes. To generalize the ideas of their work, a new type of discrete Laplace operator, which should reduce the entropy even further, is proposed in this thesis. Properties of such Laplacian are studied. It is also applied in various mesh processing techniques and results are discussed.

Introduction

Various discretizations of Laplace operator exist, differing mainly in the weights used in the discretized formula. Each of the discretizations preserves a different subset of properties (or their discrete equivalents) of the smooth Laplace operator. It can be proven, that no discretization can preserve a certain set of those properties simultaneously. This makes each discretization suitable for different purposes.

As part of this thesis, a new discretization of the Laplace operator is proposed, minimizing the lengths of differential coordinates used in the compression of dynamic triangle meshes. It is based on the assumption, that such minimization of lengths should cause a decrease of the entropy of the encoded data. It has one other big advantage over other discretizations that require the geometry information - it can be constructed from geometry of more than one mesh without requiring any complex analysis of the shapes of those meshes.

Laplace operator

Laplace operator is a second order differential operator defined as divergence of gradient. Triangle meshes are piecewise linear approximations of smooth surfaces. To apply the Laplace operator on such surface, it must be discretized. The most used discretization formula can be interpreted as a weighted sum of displacement vectors between the vertex and its neighbourhood. Applied on the whole mesh, this can be rewritten as multiplication of the matrix of positions with so-called Laplacian matrix. There exist multiple discretizations, each differing in the weights used.

Fundamental properties of the discrete Laplacian are usually described by properties of the Laplacian matrix, or the weights. The following properties are usually considered: symmetry of the Laplacian matrix, locality, linear precision, positive weights, unit sum of weights and positive semi-definiteness of the Laplacian matrix. It can be proven, that no discretization can preserve all the properties simultaneously.

Data Dependent Laplacian

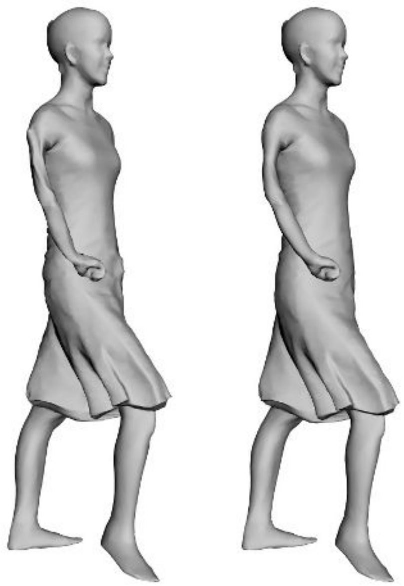
The weights of the newly proposed Laplace operator can be calculated by reformulating the discretization formula. We now desire to obtain such weights, that result in zero length differential coordinates. By solving a linear system in the least-squares sense, a Laplacian that minimizes the length of differential coordinates, is obtained. Such Laplacian is symmetric, with unit sum of weights. However, the locality is broken. The rest of the properties must be determined experimentally.

Experimental results

Experiments have shown, that unfortunately, the Data Dependent Laplacian breaks the linear precision and positive semi-definiteness. The Laplacian was also applied in mean curvature estimation, mesh smoothing, parameterization, editing, morphing and shape approximation using the least-squares meshes technique. In some of the techniques that required solving a linear system based on the Laplacian matrix, conditioning issues occurred. This resulted in visible artifacts in processed surfaces.

Visible artifacts in the shape approximation.
Left: original mesh, right: approximation

In spite of the conditioning issues, the Data Dependent Laplace operator performed reasonably well in mesh parameterization, editing and morphing. In the latter, it achieved best results from all configurations.



Mesh morphing.
Left: Tutte Laplacian, right: Data dependent Laplacian

In the case of dynamic mesh compression, the reduction of residual entropy was actually achieved. However, the conditioning issue negatively influenced the amount of distortion of the data, resulting in less effective compression than the original method.

Conclusion

A new discretization of a Laplacian was proposed. Even though it does not result in improvement in mesh compression, due to the conditioning issues, it still is quite usefull in some Laplacian mesh processing techniques.

References:

[1] VÁŠA, L. et al. Compressing dynamic meshes with geometric laplacians. In *Computer Graphics Forum*, 33, pages 145–154. Wiley Online Library, 2014.