Technical University of Košice
Faculty of Electrical Engineering and Informatics

# The application of experimental economy in the cognitive networks using Roth-Erev algorithm

**Diploma thesis**

**2018** **Bc. Marcel Vološin**

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**Faculty of Electrical Engineering and Informatics**

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**Diploma thesis**

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| Study programme: | Informatics |
| Field of study: | 9.2.1. Informatics |
| Department: | Department of Computers and Informatics (DCI) |
| Supervisor: | doc. Ing. Juraj Gazda, PhD. |
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**Košice 2018** **Bc. Marcel Vološin**

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| Thesis title: | The application of experimental economy in the cognitive networks using Roth-Erev algorithm |
| Department: | Department of Computers and Informatics, Technical University of Košice |
| Author: | Bc. Marcel Vološin |
| Supervisor: | doc. Ing. Juraj Gazda, PhD. |
| Tutor: | |
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Abstract: The thesis describes a design, development, and analysis of an agent-based NetLogo model of a spectrum trading system that uses a Roth-Erev reinforcement learning algorithm on a wholesale and various pricing mechanisms on a retail market, namely: successful-ratio, linear-reward, and trial-and-error. Model behavior is analyzed from the view of the operators, the group with the highest computational complexity of decision making, whose objective is to maximize their profit. From the simulation results can be concluded, that the operators' profit is highly affected by the *forgetting* and *experimentation* parameters. In the more advanced pricing schemes, e.g., successful-ratio and linear-reward, the addition of the *forgetting* parameter significantly improves the performance in terms of the measured indicators, namely, the average profit and Sharpe ratio. In the zero-intelligence trial-and-error approach, however, we do not see any positive changes. In addition, several notable phenomena emerged from the interactions of the agents. For example, the classification of operators who take on the role of spectrum investors as either risk-averse or risk-seeking based on the pricing schemes utilized in the retail market was shown to be an emergent feature of the model.

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| Názov práce: | Aplikácia experimentálnej ekonómie v kognitívnych sieťach využitím Roth-Erev algoritmu |
| Pracovisko: | Katedra počítačov a informatiky, Technická univerzita v Košiciach |
| Autor: | Bc. Marcel Vološin |
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Abstrakt: Záverečné práca sa zaoberá návrhom, vývojom a analýzou agentového NetLogo modelu obchodovania s frekvenčým spektrom s využitím Roth-Erev učenia posilňovaním na veľkoobchodnom trhu a trojicou cenových mechanizmov na maloobchodnom trhu, menovite: "successful-ratio", "linear-reward" a "trial-and-error". Správanie modelu je analyzované z pohľadu operátorov, skupiny používateľov s komplexným procesom rozhodovania, pri snahe o maximalizovanie zisku. Z výsledkov vyplýva, že zisk operátorov je značne ovpyvňovaný parametrami zabúdania a experimentovania. Pri použití pokročilejších cenových mechanizmov "successful-ratio" alebo "linear-reward", má zvýšenie hodnoty parametra zabúdania za následok výrazné zlepšenie sledovaných ukazovaľov: priemerný zisk a Sharpov pomer. Na druhej strane, pri použití jednoduchého mechanizmu "trial-and-error", nie je badateľné žiadne zlepšenie. V závislosti od použitých cenových mechanizmov boli v správaní modelu pozorované emergentné javy prejavujúce sa charakteristickým správaním investorov na maloobchodnom trhu.

**TECHNICAL UNIVERSITY OF KOŠICE**

FACULTY OF ELECTRICAL ENGINEERING AND INFORMATICS

Department of Computers and Informatics

# DIPLOMA THESIS ASSIGNMENT

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**The application of experimental economy in the cognitive networks using Roth-Erev algorithm**

Aplikácia experimentálnej ekonómie v kognitívnych sieťach využitím Roth-Erev algoritmu

Student: **Bc. Marcel Vološin**

Supervisor: **doc. Ing. Juraj Gazda, PhD.**

Supervising department: **Department of Computers and Informatics**

Consultant:

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Thesis preparation instructions:

1. Model the spectrum wholesale-market and retail spectrum distribution in 5G based cognitive radio networks.
2. For the modelling purposes use the reinforcement based learning mechanism and its variant called Roth-Erev algorithm.
3. Use Netlogo as the modelling environment.
4. Conduct the model parameter sensitivity.
5. Propose different pricing schemes and observe their impacts on the mutual relationship between wholesale and retail spectrum markets.

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doc. Ing. Jaroslav Porubän, PhD.

Head of the Department

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prof. Ing. Liberios Vokorokos, PhD.

Dean of the Faculty

**Declaration**

I hereby declare that the thesis is a record of original work done by me under the guidance of doc. Ing. Juraj Gazda PhD.

Košice, 26.4.2018 . . . . . . . . . . . . . . . . . . . . . . . . . .

*Signature*

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Thank you.

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# Introduction

There is no doubt about the importance of wireless communication in current society. According to Cisco's forecast [1] mobile data traffic grew sevenfold over 2016 with mobile video accounting for 78 percent of total mobile traffic (a ninefold increase is expected in the 2016-2021 period). The average smartphone will generate about 6.8GB per month by 2021 which is a fourfold increase over the 2016 average of 1.6GB with a Compound Annual Growth Rate (CAGR) of 33 percent.

Such enormous increase of data traffic and a number of active network devices changes the way society sees the frequency spectrum and arises the question about the future of current mobile communications markets that are not prepared either suitable for the observed trend as they suffer from outdated regulatory models. These models have created an oligopoly of network operators [2] which makes it extremely difficult to enter the market. A potential service provider first needs to acquire spectrum at one or more of the occasional spectrum auctions, and only then build a dedicated network or share an existing one while their new infrastructure is made operational, thus, the entry conditions are challenging.

Operator present on the market, have the exclusive right to utilize the leased frequency spectrum under the negotiated conditions. However, it is not uncommon that spectrum is not utilized evenly all the time and across the whole area where the license is valid. Majority of currently used models does not allow so-called secondary usage which enables the license holders to lease the spectrum further to the secondary users when it is not utilized.

Secondary usage in an open access network is a promising solution that may improve the utilization of the frequency spectrum and fulfill the increasing requirements and expectations of end-users. Extensive research is however required to understand the nature of such system and to design the working solution. This thesis focuses on the three-stage model of the spectrum investments in the open access network and pricing strategies of the operators as well as the interactions between the operators and end-users. The following sections of thesis describe the different approaches of spectrum trading in the open-access networks and the design of agent-based model implementing mechanisms of trading.

Analysis of the model will focus on the impact of spectrum pricing mechanisms and on the economic market indicators such as profit, its variance and Sharp ratio. Furthermore, the parameter sensitivity of the Roth-Erev algorithm will be performed to determine how the numerical results of certain models respond to changes in their inputs. By conducting a sensitivity analysis of the learning parameters, information about how well the system functions as a "unique super agent" as it looks for effective learning and pricing trends can be deduced.

The analysis of the wholesale market will explore The different working regimes (the stable mixed strategy profile, pure selection strategy and win-stay, lose-switch) determined by the variation of the Roth-Erev algorithm parameters. In turn, the effect of the data history on the profit for the operator will be examined. Moreover, operators will be characterized in terms of their spectrum investments using the stylized Sharpe ratio measure.

Research conducted during the writing of this thesis resulted in several published papers including:

- Gazda, J., Bugár, G., et al. : Dynamic spectrum leasing and retail pricing using an experimental economy. Computer Networks, 121, 173-184
- Gazda J., Tóth P., et al. : On the Interdependence of the Financial Market and Open Access Spectrum Market in the 5G Network. Symmetry. 2018; 10(1):12
- Vološin, M., Gazda, J., et. al. : Spatial real-time price competition in the dynamic spectrum access markets. 2017. In: Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) volume 10207 : EUMAS 2016. - Cham : Springer, 2017 P. 217-229.

# 1 Objectives

The main objective of the thesis is an application of the experimental economics in the model of a cognitive network with the Roth-Erev algorithm. The thesis focuses on the design and the development of the agent-based model of the cognitive network with the spectrum trading utilizing not only the Roth-Erev algorithm but also the variety of retail pricing mechanisms. Model programmed in the NetLogo environment has to be analyzed with various configurations in order to discover the emergent phenomenon that may arise from the agents' and the markets' interactions.

Not only the environment's but also the agents' characteristics have an impact on the overall model's behavior, therefore, additional effort needs to be put in their configuration.

# 2 Dynamic spectrum access networks

The arrival of 5G networks in 2020 may cause a further breakthrough in terms of a number of devices connected, bandwidth, latency and reliability required as Fig. 2.1 shows. This new generation of radio systems and network architectures will push the capabilities further than ever before by combining existing technologies with the novelty approaches to serve the variety of devices with different requirements. Authors of [3] address the characteristics of networks that need to be improved to deal with such complex task:

### High bandwidth

Observed trend of increasing data traffic requires improvement of bandwidth. The most challenging task is the improvement of an edge rate, which is the worst rate user can expect in range of the network. Typical 4G systems have capabilities to serve 1Mbps to 5% of users however 5G targets towards 100Mbps for 95%. Aggregate data rate (total amount of data network can serve) and peak rate (best data rate user can achieve) are both expected to reach about 100-1000x of current 4G networks

### Low latency and high reliability

To satisfy the requirements of virtual/enhanced reality systems, gaming industry, cloud systems etc., roundtrip latencies need to be reduced from the current 4G's 15ms as close as possible towards a 1ms to support devices dealing with time-critical problems for example communication of autonomous vehicles which does not only require low latency of communication but also highest possible reliability of network to be able to operate in environment of crowded cities without a risk.

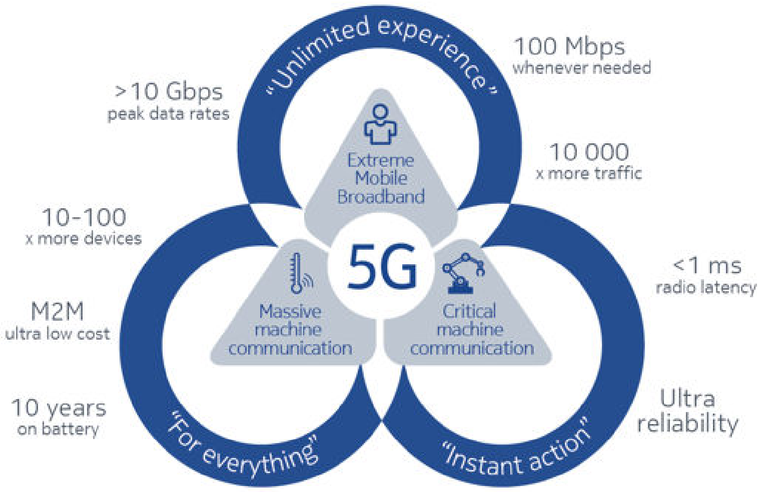



Figure 2.1: Goals of 5G networks according to Nokia [4]

### Massive machine communication

5G networks also need a native support for machine-to-machine (M2M) communication which is expected to grow in the near future. Macrocells are therefore required to serve up to $10^4$ of various devices such as metering sensors, smart grid components, wearable devices etc., at the same time. According to [5], current systems are able to easily serve 5 devices at 2Mbps each, however, are not optimized for short data blocks and therefore cannot serve 10000 devices each requiring 1Kbps. Enhancement of low data rate, on the other hand, can not degrade performance required by traditional high-rate mobile users.

### Low energy and cost

The efficiency of the 5G network represented by Joule per bit needs to be improved at least 100x to keep service costs on par with the 4G. This can be achieved via the use of small cells that are not only 10-100x cheaper but also more power efficient than macrocells. Another way of decreasing the costs is the utilization of mmWave spectrum, ranging from 30 GHz to 300 GHz, that is 10-100x cheaper per Hz than spectrum below 3GHz. However, usage of this spectrum requires new techniques to deal with high propagation loss, directivity and sensitivity to blockage [6].

Although, 5G network seems to improve every important aspect of wireless communication it primarily does not solve the major issue which is the scarcity of

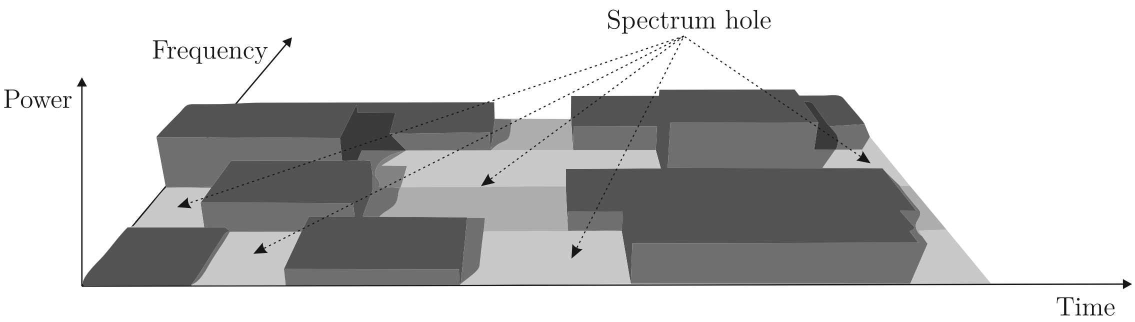



Figure 2.2: Spectrum holes - Temporarily unused spectrum

frequency spectrum. Importance of this issue is being increased, as the demand for spectrum grows. China with it's 1.286 billion of mobile subscribers, which makes it the country with the largest number of mobile users on the planet, is a great example of the country facing a spectrum scarcity. Sufficient frequency bands needed for 5G can be divided into two parts: bands below and above 6GHz. While finding an unoccupied spectrum above 6GHz is not an issue, vital bands below 6GHz, needed for their coverage capabilities, are heavily utilized by various services. According to authors of [7], spectrum required by 5G in China is significantly larger than currently required by 2G, 3G, and 4G networks, therefore using only licensed spectrum may not be sufficient, which means that sophisticated mechanisms used in so-called cognitive networks are required to meet 5G requirements.

The term "Cognitive network"(CN) has multiple definitions. According to Thomas et al. [8] CN can be described as a flexible and extensible framework able to, directly and indirectly, observe a network in order to gain the information used to alter a future behavior depending on the state of the network. Its goal is to predict the conditions, to act proactively rather than retroactively in order to improve end-to-end performance under which better spectrum utilization, quality of service (QoS) and security is considered. Implementation of the cognitive network requires more effort and a more complex system than noncognitive one, however, these costs will not be considered substantial in the near future with high probability.

Principles of spectrum auctions are not only insufficient for the future high data traffic networks but also guilty of spectrum holes emergence in the frequency

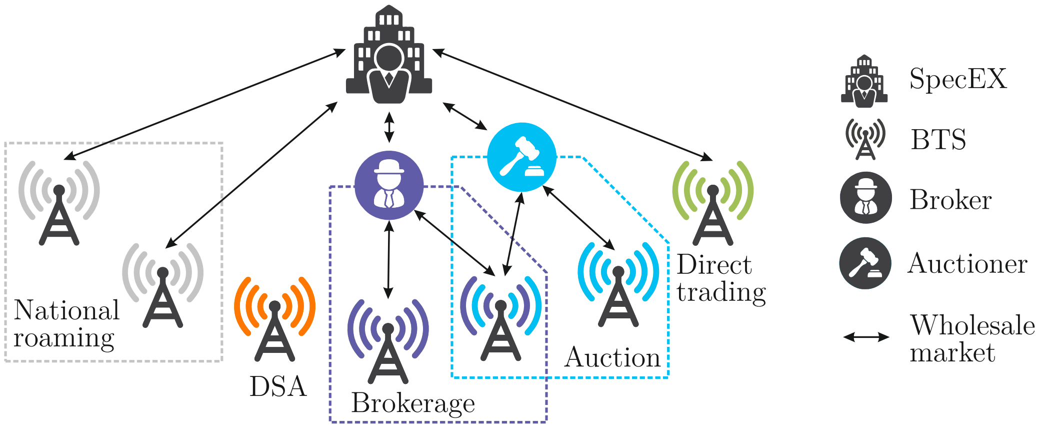



Figure 2.3: Different approaches of wholesale market mechanisms

spectrum (Fig. 2.2). These temporally unused chunks of the licensed spectrum can be utilized under certain conditions but this secondary usage is prohibited according to the policies of most currently used regulatory models.

A promising solution to promote increased competition is the establishment of the *open access* network concept [9]. An open access network is a public-private partnership that is structured similarly to those in the restructured electricity markets, which have been in operation for the past twenty years. Service providers in the open access network compete for frequency resources on a wholesale market and then sell those resources to mobile communications end-users on a retail market. The regulators hope to increase spectrum utilization and stipulate the level of operator competition through *dynamic spectrum access* (DSA), *end-user multi-homing* techniques and *national roaming*.

## 2.1 Wholesale market models

Dynamic spectrum access (DSA) also referred to as dynamic spectrum management (DSM) is a paradigm that allows utilization of unused frequency spectrum, spectrum holes, by so-called secondary users in the licensed spectrum bands [10]. According to [11] it may also decrease costs associated with the spectrum transactions and entry barriers that potential operators face too, however, implementation requires high investments. This promising approach is a subject of ongoing research on both low and high levels.

On the low level, the interference detection and its avoidance is the main issue [12], because the QoS of primary users cannot be diminished by secondary usage. Dynamic spectrum access requires sensing of ongoing utilization with the aim to not only discover white holes but also predict their future appearance which is the especially challenging task when the communication patterns of license holders are not defined.

### 2.1.1 National roaming

According to [13], national roaming is mostly considered to be an instrument of public policy, with the ability to increase competition by allowing the entrance of new competitors to the market without whole infrastructure already built. It is especially suitable for the countries with the large area and rather insufficient coverage. In case of emergency situation, end-users are most likely to appreciate the existence of this roaming because thanks to its implementation, it is possible to perform emergency calls with the phones with or even without SIM inserted over the foreign network all over the European Union and the USA (with few exceptions [13]). The mechanism itself improves the allocative efficiency (network capacity and spectrum utilization) by increasing served demand and decreasing the unused capacity. On the other hand, this approach meets with the resistance of established operators holding the monopoly and their objections against assistance to potential competitors. The [11] also reminds that national roaming requires careful regulation due to a possibility of agreement about the high roaming fees among operators that will be charged together with retail prices.

### 2.1.2 Opportunistic sharing

Opportunistic sharing is an approach which uses the advantages that cognitive networks offer. Spectrum holes discovered by sensing of real-time utilization of the licensed spectrum, can be used opportunistically by non-legitimate users [14], which increases the spectrum utilization. However, attention to retention of the interference-free system is required because sharing the same radio channel may degrade the system performance and limit the primary user. One of the possible solutions is the application of transmit beamforming.

### 2.1.3 Secondary trading

On the high level, secondary spectrum trading mechanisms are the subjects undergoing intense study. These determine the actions taking part in process of trading and define the market structure. Authors of [15] describe following mechanisms:

#### Auction

In the auctions, stakeholders are divided into two groups with different interests, selling brokers with their goal to maximize revenue by trading and buying and service providers who attempt to acquire bandwidth at the desired price. As was already described, auctions are the way the operators negotiate with the state authorities to obtain the spectrum license, however, licenses acquired this way are long-term, mostly with duration of 10 years. Auction mechanisms proposed for the usage on the secondary markets are considered to operate in shorter than hour intervals offering the licenses with corresponding time validity.

On the other hand, the nearly real-time auction process is not trivial since interference-free connections need to be ensured in the networks. Authors of [16] claim that many of the existing auction mechanisms require complex bid expression that grows exponentially with the size of goods, therefore, are insufficient due to their inability to deal with NP-hard problems in real-time. According to [15] there is also a possibility of monopoly in the auction controlled spectrum trading environment, with few participants able to overbid the concurrency easily.

#### Brokerage system

A process of trading in a brokerage system is managed by a broker, who receives the offers from primary users and secondary users and afterward matches the pairs to optimize the social welfare. Broker plays a role of mediator that attempts to satisfy both sides of a contract by maximizing the subscribers' surplus and providers' profit. This approach is commonly used in smart grid models [17] [18] including the European Union's Horizon 2020 founded project NOBEL [19].

### Direct trading

Unlike in the brokerage system, in a direct trading, there is no mediator between the sellers and buyers on the secondary market. The major issue in the direct spectrum trading is a pricing. Authors of [15] explain, that a trading price is set according to the demand and the supply. In case of high demand and low supply prices are set by sellers and on the contrary when demand is lower than supply, prices are set by the buyers who turn over each offered price in mind.

## 2.2 Retail market approaches

Network switching allows users to automatically choose the right network to get the best connection according to given requirements. Price, throughput, latency etc. or combination of given can be a deciding criterion. This mechanism does not require a support from the operator. The approach of end-user switching was applied in Project Fi by Google [20] which represents a mobile virtual network operator (MVNO). Fi-ready phones can seamlessly switch to the fastest of three 4G LTE networks on the area of USA (Sprint, T-Mobile, and U.S. Cellular), and get free access to 1 million Wi-Fi hotspots to maintain the best possible connection in any location with the real-time network switching. To ensure the security, Project Fi supports the encryption of communication through an automatic VPN over Wi-Fi hotspots.

End-user multihoming is an approach that allows users to utilize multiple mobile networks at the same time to increase connection throughput by aggregating network capacity. Data packets are being split by the user's device and no information about an adoption of this mechanism is shared with concerned networks, so no operator support is required in this user-centric strategy. Fig. 2.4 shows an example of multihoming between the end-user and various service providers' devices. End-user multihoming can be described as a special case of network switching when not single but at least two best networks are chosen to be utilized.

Multihoming exists in three forms depending on who controls the traffic flow. We distinguish between the network-centric and user-centric approach which combined creates hybrid option [21]. This approach performs well in the environments with a low density of network devices, however, according to [22] it has no benefits over switching in dense environments. Results of simulations show that

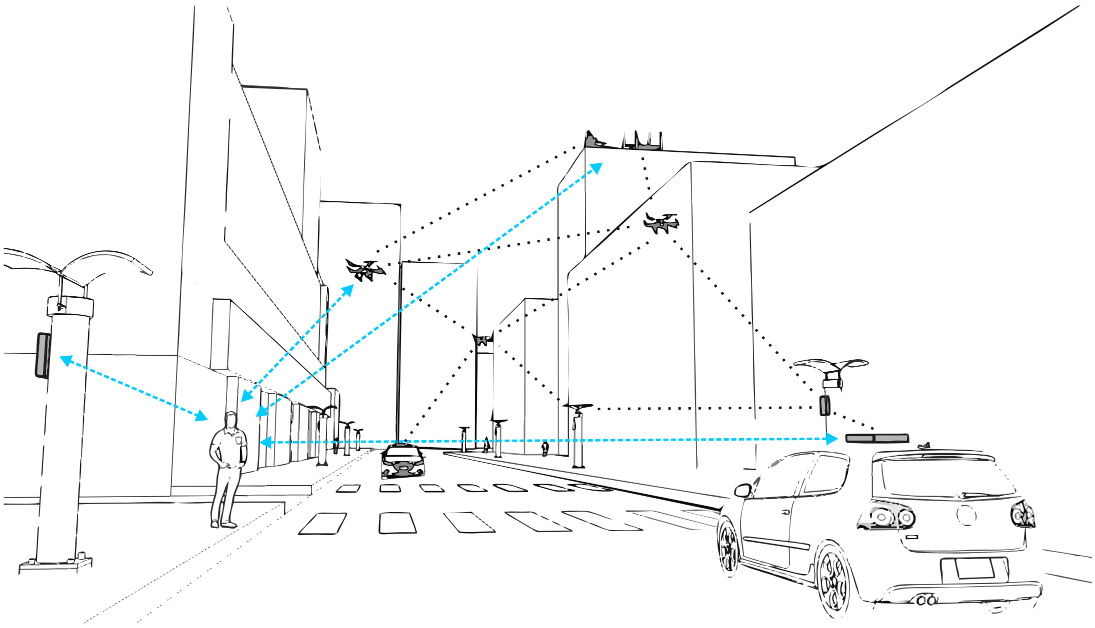


Figure 2.4: End-user multihoming between user, WiFi access point, rooftop base transceiver station (BTS), dron mounted BTS and vehicle mounted BTS

usage of a best near network is more efficient than splitting the packets between more networks in the most cases.

Devices with the multihoming support have to be equipped with the multiple network interfaces to efficiently increase their throughput by using parallel transmissions over multiple paths [23]. Because different paths tend to have different bandwidth and delay, mechanisms to mitigate out-of-order packet receiving are required. Another issue related to the multihoming is a need to monitor and analyze paths to ensure the optimal routing. The scope of the thesis is the economic analysis of network featuring end-user switching with a spectrum trading capabilities thus none of the given issues will affect the analyzed model.

# 3 Modeling approaches: game theory vs agent based modeling system

Game theory is a collection of analytical tools made to help understand the decision making of rational entities who take their knowledge or expectations of other decision-makers' behavior into an account. Various highly abstracts models that represent the classes of real-life situations exist. The most famous are: the theory of Nash equilibrium (used to study oligopolistic and political competition), the theory of mixed strategy equilibrium (explains the distributions of tongue length in bees and tube length in flowers), the theory of repeated games (used to study social phenomenon like threats and promises) and the theory of the core (describes a sense in which the outcome of trading under a price system is stable in an economy that contains many agents) [24]. Usage of game theory allows the investigation of a steady state existence and convergence to the steady state.

On the other hand, according to [25] game theory struggle to model the real intermediated competitive markets that are characterized by power-law distributed market shares. The more competitive the market is, the larger the magnitude of the exponent in the power law equation is. Therefore game theoretic models may struggle to capture the dynamic process of interactions among dense networks of meta-stable agents precisely.

Another issue of this modeling approach is its inability to model the entities with a bounded rationality e.g. asymmetries in abilities and in perceptions of a situation by different entities[24]. Game theory assumes that the entity's knowledge of the rules is perfect and its ability to analyze the situation is ideal. This drawback, however, makes it impossible to model the desired spectrum trading system with the non-cooperating operators and end-users because the operators are not supposed to share their information therefore are required to operate with

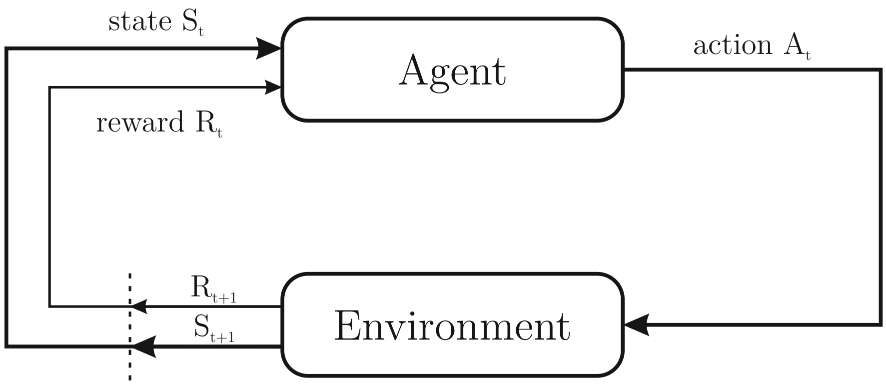



Figure 3.1: Agent's interaction with the environment in an agent-based model

the only partial knowledge about the environment state.

The second well-known approach is the agent-based modeling (ABM) commonly used to simulate the behavior of systems that consist of a large number of subjects interacting in a shared environment. These models are the systems of identifiable, discrete and self-directed individuals with own characteristics, rules, behavior and decision capability. Agents are situated in the environment with which they can interact to achieve the desired goals. The behavior of agents is commonly flexible which allows the process of learning driven by the previous experience. Figure 3.1 shows a simple process of interaction between the agent and the environment. The inner state of the agent determines its actions towards the environment. Each action is rewarded which serves as a feedback that affects the future behavior through the change of inner state or modification of behavioral rules [26]. Agent's actions may affect the environment and subsequently other agents or the other agents directly.

Agent-based models are popular because of their modularity, flexibility, expressiveness ability to capture emergent phenomena and the possibility to be executed in parallel. Another advantage is the ability to be coupled with different type of models including, continuum models or models of human response to simulate specific or very complex scenarios [27], for example pandemics [28], pedestrians [29], evacuation [30], ocean microbe ecosystems [31] etc.

These models are not only used by researchers and modelers to confirm their assumptions but also in real situations by decision-makers. An integrative decision support system DANUBIA [32] is an example of the agent-based simulation model used for decision support with the components for natural science as well as socio-economic processes and their interactions. It was used for simulation of various sectors including e.g. farming, economy, water supply companies, private households, tourism etc., with the intention to predict the climate change in the area of the Upper Danube. The UrbanSim [33] is another great example of an agent-based simulation system. It is used for planning support, analysis of urban development, quick evaluation of land parcels or building suitability for affordable housing development mainly in San Francisco. It also simulates the interactions between land use, transportation, the economy, and the environment. To prevent the potential San Francisco's housing crisis a number of affordable housing units was estimated by the simulations made in UrbanSim's sub-system called Penciler.

There are however some issues related to the ABM too. It is very important to determine the right amount of abstraction so the result will serve its purpose well. Another issue is a performance. Even trivial models may become too demanding with a plenty of agents involved, therefore it is advised to reconsider the usage of ABM approach. Moreover, due to potential irrationality or subjectivity of humans' behavior in the models with the human agents involved, one must be careful when implementing such model to capture the nature of decision making [34].

The ABM approach is commonly applied by researchers investigating smart-grid electricity distribution networks. For example, in [35] and [36], the strength of the ABM approach was demonstrated by modeling a self-healing grid through collaborative fault location and power restoration. As was highlighted in [37], there are several points of commonality between smart-electricity distribution and real-time spectrum distribution. However, one key difference is that wireless communications networks are much simpler. This is because electricity markets have resources that are expensive to turn on, which limits the speed with which they can be adjusted. In contrast, wireless network elements are quick to respond and can be efficiently controlled with marginal prices. These underlying realities make the application of ABM of special relevance in real-time spectrum trading schemes. A class of models designed to assess the efficiency of spectrum markets has been successfully implemented in several cases, e.g., increased spectrum utilization by allowing the incumbent users to further lease the spectrum to other entities [38], the formation of coalitions to improve spectrum sharing access [39], the efficiency of brokerage mechanisms, an auction-based approach with direct trading [40], and efficient tax policies that impact the market economy [41]. Finally, the market dynamics, including the transactional and switching costs for mobile markets based on ABM, are evaluated in [42].

# 4 The proposed agent based model of the wholesale and retail market spectrum distribution

The proposed agent-based model focuses on the competition of operators with a variable capacity and coverage who compete to serve a common pool of end-users. In these scenarios, operators dynamically lease spectrum from a Spectrum Exchange (SpecEx) server and then compete to sell the spectrum to end-users in order to maximize their individual profits. Our intent is to understand how the operators evaluate their spectrum investments on the wholesale market and, in turn, make their selling decisions on the retail market, respectively. Participating operators' heterogeneity arises from the different placements of base transceiver stations (BTSs) in the region of interest, which results in various spectrum coverage efficiencies.

The model implements a three-stage agent-based computational model to study the spectrum investments and pricing strategies of the operators as well as the interactions between the operators and end-users. In Stage I, the operators lease temporarily varying frequency resources from the SpecEx server. Owing to the uncertain demand in the retail market, a heuristic probabilistic model based on the Roth-Erev algorithm is applied [43]. It resembles the natural learning behavior in smart-electricity distribution studies, e.g., [44], [45]. The combined use of the two learning parameters (*forgetting* and *experience*) in the Roth-Erev algorithm reflects the fact that some information needs to be stored for a period of time, but other information should be forgotten at certain time horizons. A similar philosophy is used when processing financial market data, where traders seek to optimize the extent to which the market history is important for forecasting. In general, the

Roth-Erev algorithm can be perceived as the product of a framework coined under the term *experimental economy*. In Stage II, operators simultaneously announce their retail prices to users using a family of well-known pricing strategies. In Stage III, users jointly make their decisions based on the prices and level of quality of service (QoS) offered by the operators. In our analysis, it is assumed that the end-users are equipped with the software-defined radios and can transmit in a wide range of frequencies as directed by the operators. Such network structure is advantageous because it places most of the implementation complexity for dynamic spectrum leasing and allocation on the operators, and thus is easier to implement than a "full" cognitive radio network.

This study focuses on DSA and *end-user multi-homing*, both of which have great potential to improve the economics of the operators, such as the profit, profit variance, etc.

## 4.1 Model preliminaries

The simulation environment was inspired by models proposed in [46] and [47]. Operators' base stations are uniformly distributed across the linear region along with the end-users according to the assumptions of Hoteling competition model [48], not taking the typical travel costs considered in the original Hoteling model of spatial market competition into the account. One dimensional environment may look like an inappropriate simplification of the real-world situation but we believe it is sufficient for the objectives of this research. Uniform distribution of agents along the line segment with a single BTS in the middle can be easily transformed into two-dimensional arrangement since the exact coordinates of agents does not affect the agents' behavior, but the distance between agents does. The tested scenario with multiple base stations is especially suitable for investigation of the competition between entities with different properties and environmental advantages. Operators with the base stations positioned on the sides of Hotteling's linear environment may need to compensate the disadvantageous location via slightly decreased prices to attract more users.

Figure 4.1 illustrates the agent arrangement in the simulated world. Triangles represent the end-users and colorful arrows are the base stations of five different operators. The green tint shows the overlapping coverage of base stations how-

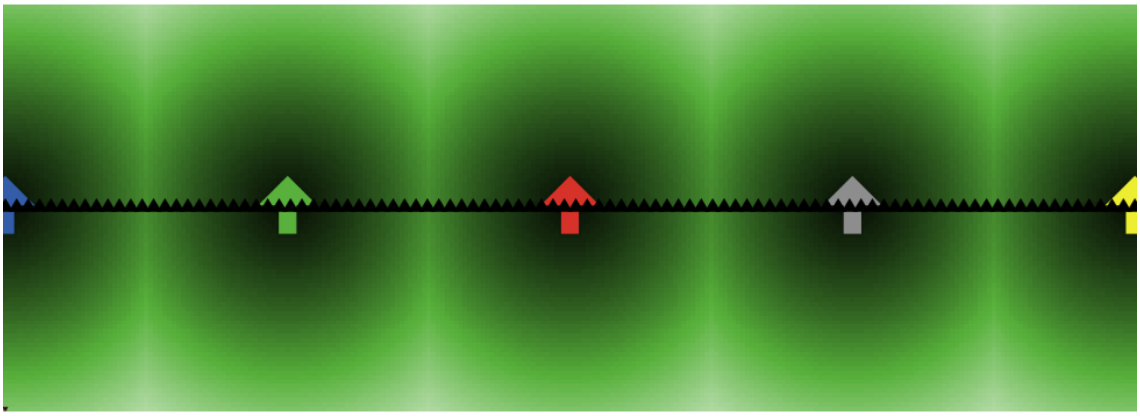


Figure 4.1: Linear deployment

ever in our simulated environment there is no interference between the overlapping effective regions of the networks of different operators, since we consider an interference-free system in which the spectrum allocations are valid throughout the region and no two sessions can occupy the same frequency band, as was proposed in [49]. The basic motivation for competition between spatial operators is found in the work provided in [50] in which the authors study how two operators simultaneously compete in the case where one provider operates in a sub-area of the other. In our scenario, one operator maintains its coverage in a sub-domain of four other operators, which is a highly competitive scenario.

Figure 4.2 illustrates the basic hierarchy of the agent in the model. Three different agent types are present in the simulation environment, each with its own interests and goals. The role of SpecEx server is rather simple - lease frequency spectrum for the fixed price on the wholesale market. It is important to clarify, that the SpecEx server used for the purposes of the simulations controls spectrum large enough to satisfy the needs of all operators present, therefore operators do not compete for the resources via overbidding the competitors. Competition between operators takes place on the retail market only, where the overlapping coverage of the BTSs enables end-users to choose the service provider based on their preferences.

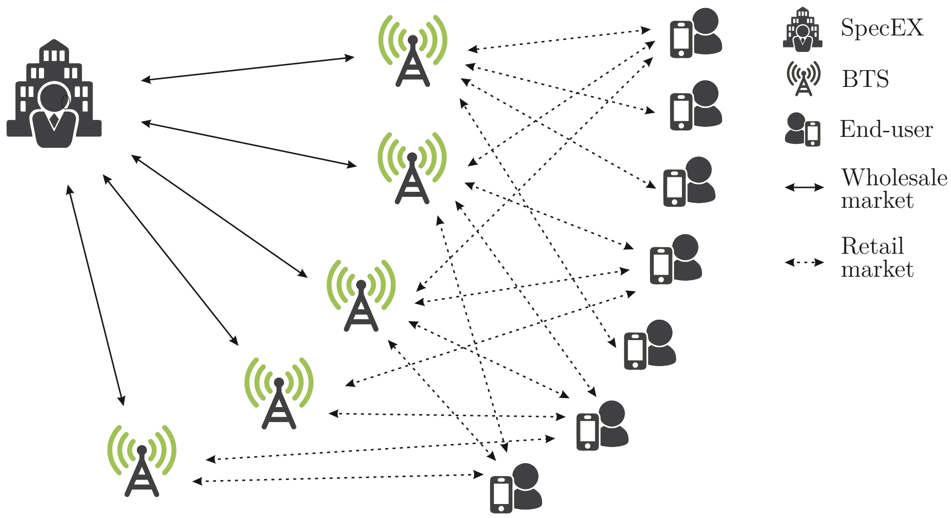



Figure 4.2: Structure of agent-based model

## 4.2 Wholesale market trading

Mechanism of the implemented wholesale market is inspired by the methodology of adaptive heuristics commonly used in electricity networks. Authors of [44] and [45] used the principles of reinforcement learning to learn the behavior patterns of electricity networks in order to optimize retail-prices with the aim to increase profit. At their core, adaptive heuristics are simple behavioral rules that are directed towards payoff improvement but may be less than fully rational. In the model, two well-known heuristic based methods were applied. First, the cumulative payoff matching strategy in which the agent initiates actions in the next period with probabilities proportional to the cumulative rewards they collected in the past [51]. Second, the Roth-Erev algorithm was applied, which adds the forgetting and experimentation parameters to the action probabilities.

The SpecEx server offers frequency channels on the wholesale market. The $j$-th operator leases $i$ frequency channels at each time instant $t$, $(1 \leq i \leq N)$. The $N$-size vector is defined as $\mathbf{v}_j$, which symbolizes the possible actions of the operator on the wholesale market i.e. the number of channels being leased by the $j$-th operator. In the model the cardinality of the action space of the operator is equal, i.e., $\forall j$, $||\mathbf{v}_j||_0 = N = 30$.

For simplicity, the index $j$ is ignored in the following expressions. The algorithm maintains the propensity $q_i(t)$ for each possible choice of operator, i.e., for $\mathbf{v}(i)$. By considering the idea underlying the Roth-Erev algorithm regarding *experience* and *forgetting*, can be defined as:

$$(\forall i)\ :\ q_i(t+1) \quad = \begin{cases} [1-r]q_i(t) + E_i(e,n,k,t)\,, \\ \qquad \text{if}\ \ (\,[1-r]q_i(t) + E_i(e,n,k,t)) \geq 0\,, \\ \\ 0\,,\ \text{else} \end{cases} \tag{4.1}$$

$$(\forall i)\ :\ E_i(e,n,k,t) \quad = \begin{cases} \Pi_k(t)[1-e] & ,\ \text{if}\ i = k\,, \\ \Pi_k(t)\frac{e}{N-1} & ,\ \text{if}\ i \neq k\,, \end{cases} \tag{4.2}$$

where $q_i(0)$ is the initial propensity of action $i$ at time $t = 0$, i.e., aspiration level, $q_i(t)$ is the propensity of action $i$ at time $t$, $\Pi_k(t)$ is the profit obtained for taking action $k$ at time $t$, which in the model is interpreted as the difference between the revenue achieved on the retail market and the total cost for the spectrum resources on the wholesale market. The parameter $r$ is the *forgetting* parameter, $e$ is the *experimentation* parameter, and $N$ is the number of actions the operator must choose. The *experimentation* parameter $e$ where $e \in [0,1]$ assigns different weights between the played action and the non-played actions, which in turn influences the redistribution of profits along the propensity vector $\mathbf{q}$. The *forgetting parameter* $r$ where $r \in [0,1]$ contributes toward an exponential decrease in the effect of past results.

The propensities are then normalized to determine the probabilistic action selection policy for the next wholesale market round:

$$p_i(t+1) = \frac{q_i(t+1)}{\sum_{j=0}^{N-1} q_j(t+1)}. \tag{4.3}$$

The above equalizations postulate the basic principles of the Roth-Erev algorithm, which is commonly used across many heuristic-based learning problems, including the problem of modeling electricity markets. In general, the Roth-Erev algorithm is used to solve a myopic stimulus-response problem in the following form: given the profit of the selected strategy, what strategy should I choose next? In answering this question, concrete look-ahead reasoning is considered, e.g., while considering the potential impact of own actions on the choices of other operators in the future. This is in full agreement with the bounded rationality assumption of the agents present in the simulated environment.

In a discussion, a special model configuration is employed when $e = 0 \wedge r = 0$. The decision policy of operator can be formulated as follows:

$$(\forall i) : q_i(t+1) = q_i(t) + \Pi_i(t). \tag{4.4}$$

In this case, no experiments are conducted in the strategy space and past values are not discounted. This special case characterizes the cumulative payoff matching rule and was initially proposed by psychologist Nathan Herrnstein [52]. Later, the application of this rule spread to the wireless communications arena [53]. The key feature of the algorithm is that the probability of choosing an action increases monotonically along with the total payoff generated in the past.

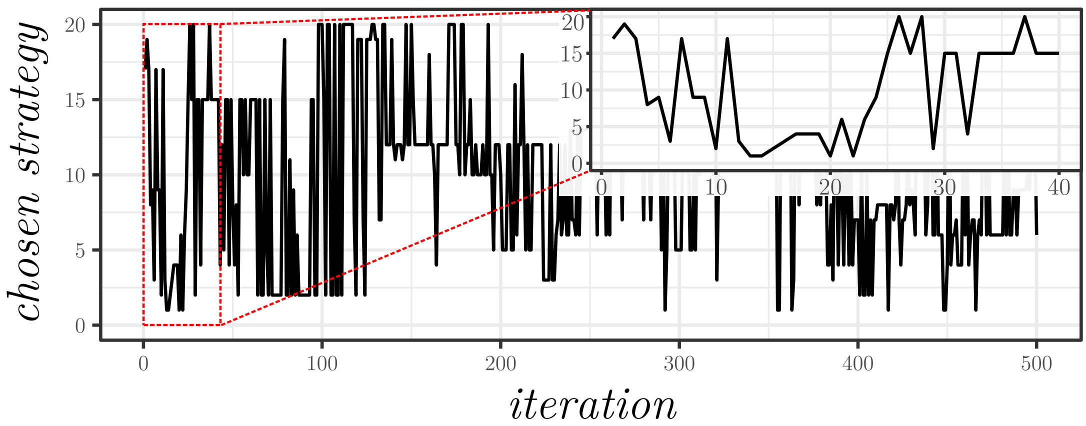



Figure 4.3: Roth-Erev - example $e = 0.2$ $r = 0.2$ and initial $q = 0.5$

Figure 4.3 shows an example of strategy selection controlled by the Roth-Erev algorithm based on randomly generated rewards from interval $< 0; 1 >$. Following parameters were chosen for the illustration purposes: $e = 0.2$ $r = 0.2$ and initial propensity of all strategies $q = 0.5$ which results in probabilities being all equal in the initial state Tab. 4.2a). During the process of learning, the mechanism adjusts the propensities and therefore probabilities according to rewards they obtain as shown in Tab. 4.2. In the first iteration, a random strategy is chosen out of 20 possible options. The selected strategy number $17$ is rewarded by $0.25$ which increases the future probability of choosing the concerned option as seen in Tab. 4.2b) and decreases of probabilities of other strategies at the same time. Change of the probabilities is affected by chosen experimentation parameter $e$, recency parameter $r$ and initial propensities $q$. In the second iteration, strategy number $19$ is chosen and rewarded by a slightly lower reward of $0.15$. Tab. 4.2c) shows that probability of strategy $19$ that earned lower profit has a lower probability of being chosen in the next iteration than strategy number $17$. In the next iteration strategy number $17$ is chosen again obtaining the reward of $0.30$ which results in a further increase of its probability meanwhile probabilities of other strategies are lowered according to algorithm rules. In the next iteration strategy number $8$ is rewarded by $0.35$ which adjusts the probabilities in the way shown in Tab. 4.2e). Tab. 4.2f) shows the state of probabilities after series of 500 decisions that were rewarded by random rewards from interval $< 0; 1 >$. Due to the experimentation parameter $e$ being greater than $0$ all strategies retain at least minimum probability of being chosen.

## 4.3 Retail market trading

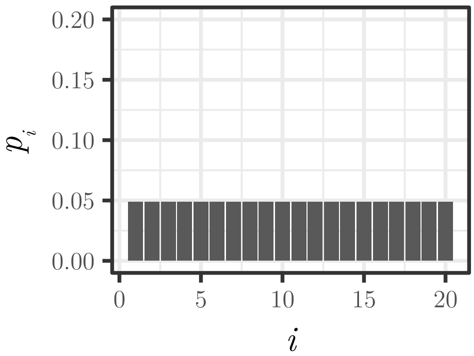
The concept of the retail marked used in the model was inspired by the micro-economics theory [38]. QoS seeking end-users evaluate each offer proposed by the operators according to own needs to determine the most suitable one. QoS is determined by utility function $U$ which is affected by the spectral efficiency of the end-user-operator pair. Utility function is assumed to map quality related-parameter $r$, where $0 \leq r \leq \infty$, onto an interval of real numbers. The spectral efficiency of the transmission between the $i$th end-user with respect to the $j$th operator is defined as:

$$r_{i,j} = \log_2 \left[1 + \frac{P_s}{N_0}\left(\frac{d_{i,j}}{L/4}\right)^{-2}\right], \tag{4.5}$$

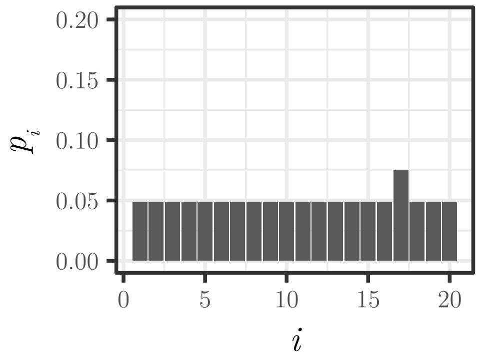
where $N_0$ is the additive white Gaussian noise (AWGN) variance, $P_s$ is the signal power, $d_{i,j}$ is the distance between the $i$-th end-user and BTS of the $j$-th operator. $L$ represents the total length of the linear region ($L = 1,000m$ in used simulation setup). Setting $P_s = 2N_0$, guarantees the end-user signal-to-noise ratio (SNR) $SNR = 3dB$ at a distance of $L/4 = 250m$ from the BTS of the operator.

The utility of the $j$th operator's offer towards the $i$th end-user is defined as the mapping $U_{i,j} : \mathbb{R}_0^+ \rightarrow [0, 1]$:

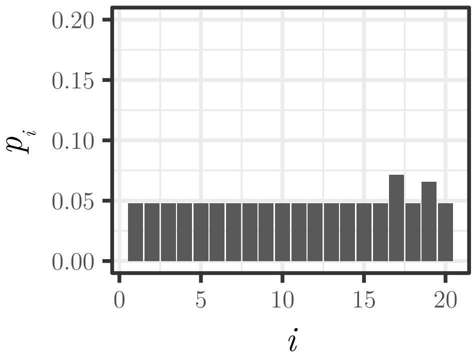
$$U_{i,j} = e^{-\alpha(\frac{1}{r_{i,j}})^{\beta}}. \tag{4.6}$$



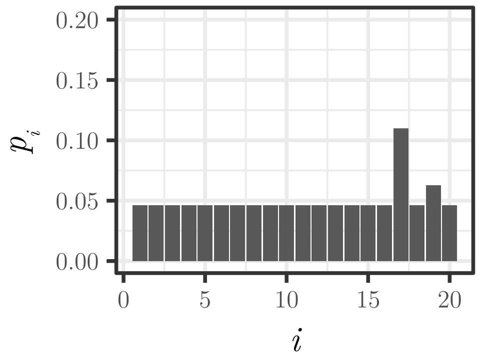

(a) Initial state



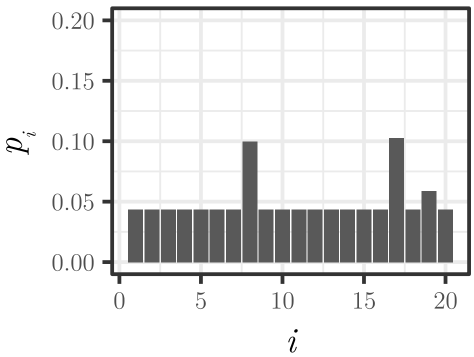

(b) After strategy 17 earns reward 0.25



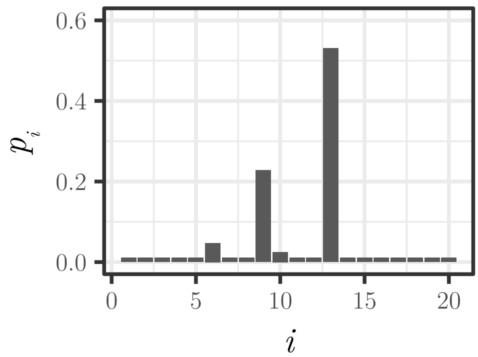

(c) After 19 earns 0.15




(d) After 17 earns 0.30




(e) After 8 earns 0.35




(f) Probabilities after 500 iterations

Table 4.2: Roth-Erev - Evolution of probabilities

where $\alpha, \beta$ are positive real-valued parameters. Higher spectral efficiency results in a higher utility $U$ of connection. A rational expectation of utility is a general concept that is the basis of modern finance. Accordingly, each end-user has a utility function, which is an element of the decision-making process and determines

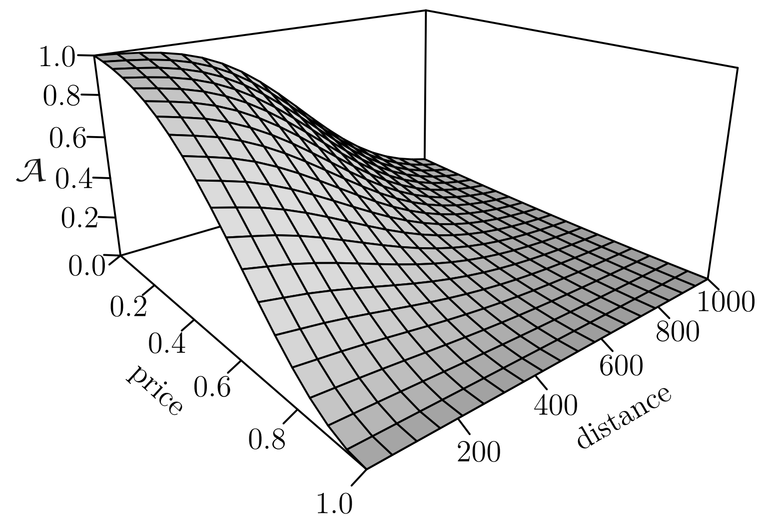



Figure 4.4: Acceptance probability of the end-user

their preferences. The proposed model uses the micro-based Cobb-Douglas production formula for $U_{i,j}$, which historically belongs to the new Keynesian models [54]. It reflects the dual nature of the relationship between end-users and operators. On the one hand, it is related to the spectrum efficiency level, on the other hand, it is closely linked to end-users.

QoS, however, is not the only decisive factor in offer evaluation. The operators purchase a limited and variable number of frequency channels on the wholesale market and then seek to maximize their total revenue by reselling those resources on the retail market. Thus, it becomes necessary to measure the utility of the end-users while analyzing the role of pricing from the perspective of the operators. Here, the perception of the service for end-users is remarkably different if the price has increased or been reduced. In practice, end-users are satisfied with the service if both the quality and price are considered acceptable [55].

Therefore, it seems reasonable to enforce an acceptance probability $\mathcal{A}_{i,j}$ which depends on QoS (through the utility $U_{i,j}$ defined by Eq.(4.6)) and the price $\kappa_{i,j}$. The acceptance probability that the $i$-th end-user will accept the offer from the $j$-th operator is a function of the $\kappa_{i,j}$ and $U_{i,j}$ variables, and is defined as a mapping of $\mathcal{A}_{i,j} : [0,1]^2 \to [0,1]$, which can be expressed as:

$$\mathcal{A}_{i,j}(U_{i,j}, \kappa_{i,j}) = 1 - e^{-cU_{i,j}^{\delta}(1-\kappa_{i,j})^{\gamma}}, \tag{4.7}$$

where $\delta, \gamma \geq 0$ are the parameters describing the sensitivity of the end-user to both

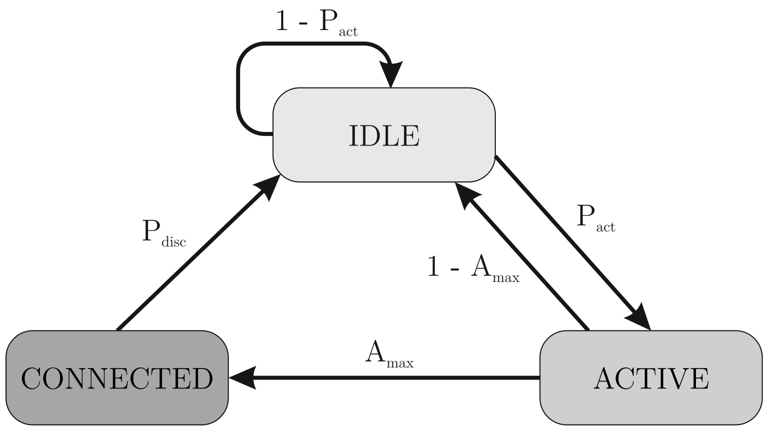



Figure 4.5: Switching of End-users

the utility and price, respectively. This particular acceptance-probability function was selected based on its relationship to the Cobb-Douglas demand curves used in microeconomics [54]. Figure 4.4 shows the nonlinear dependency of the distance $d_{i,j}$, price $d_{i,j}$, and acceptance probability $\mathcal{A}$. As can be seen, despite its highly nonlinear characteristics, intuitive assumptions are confirmed. When the end-user is infinitely close to the BTS of the operator and simultaneously the price $\kappa \to 0$, the end-user tends to accept the offered service with $\mathcal{A} = 1$. On the other hand, there is a strong nonlinear decay of the acceptance probability $\mathcal{A}$ as both the price and distance increase. Dependency between the price, utility, and acceptance probability is in general governed by the setting of the $\alpha$, $\beta$, $\gamma$, and $\delta$ parameters.

End-users present in the model can switch between 3 different states namely: $IDLE$, $ACTIVE$ and $CONNECTED$ in a stochastic way. Fig. 4.5 explains the nature of switching. State change $IDLE \Rightarrow ACTIVE$ occurs randomly with the probability of $P_{act}$. When in $ACTIVE$, the user willing to use a wireless service searches for the offer with maximum $\mathcal{A}$ by evaluation of all offers found. If the offer is accepted, end-user switches its state from $ACTIVE$ to $CONNECTED$, or switch $ACTIVE \Rightarrow IDLE$ else. The user remains in $CONNECTED$ state, using the service, with the probability of $1 - P_{disc}$. However, it is important to mention, that only single state change of end-user is possible per retail market iteration (principle of simulation iterations will be described in later a section).

## 4.4 Pricing schemes

The main issue of the retail market included in modeled DSA network is the pricing, that provides a reasonable profit for the operators. As a result of the end-users' *multi-homing* and the role of an acceptance probability in the decision-making process, operators may benefit from real-time dynamic pricing schemes, which means that the price of a product or service can vary over time. Generally speaking, the aim of dynamic pricing is to increase the profit of the operator. Modeled end-users are both quality and price sensitive, which requires operators to take advantage of different price intelligence mechanisms to attract the consumers' demand.

In this section, three typologically different retail pricing mechanisms will be reviewed. The trial-and-error (T-E) strategy belongs to the group of zero-intelligence learning models, while the linear-reward-inaction (LR-I) strategy is based on the learning-automata in which the automaton selects its current action based on past experiences in the environment. In contrast, the successful-ratio strategy (S-R) determines the price based on the instantaneous technical indicators of the network, i.e., the number of end-users accepting the current price, rather than the market characteristics (instantaneous or cumulative profit). Implementation of multiple pricing schemes makes thorough agent-based model analysis possible and creates an opportunity to discover more emergent phenomena.

### 4.4.1 Trial-and-error strategy

When the spectrum demand functions are unknown and vary over time, one possible solution how to optimize the revenue of the operator is to continuously adjust the spectrum price based on the observed instantaneous profit. A simple method to achieve this is the T-E strategy proposed in [56]. In this method, an initial price is chosen randomly from the range $(0, 1]$. At regular intervals with a small probability, a random price increment is chosen from a truncated normal distribution that has a small $\sigma$. After the price changes, the profit earned in the next time interval is monitored. If the profit of the operator is improved after the new price is adopted, that price is used. Otherwise, the operator reverts back to the previous price and the whole process continues. Figure 4.6 clearly shows that Trial-and-error pricing strategy with $\sigma = \pm 0.05$ is the most stable mechanism implemented

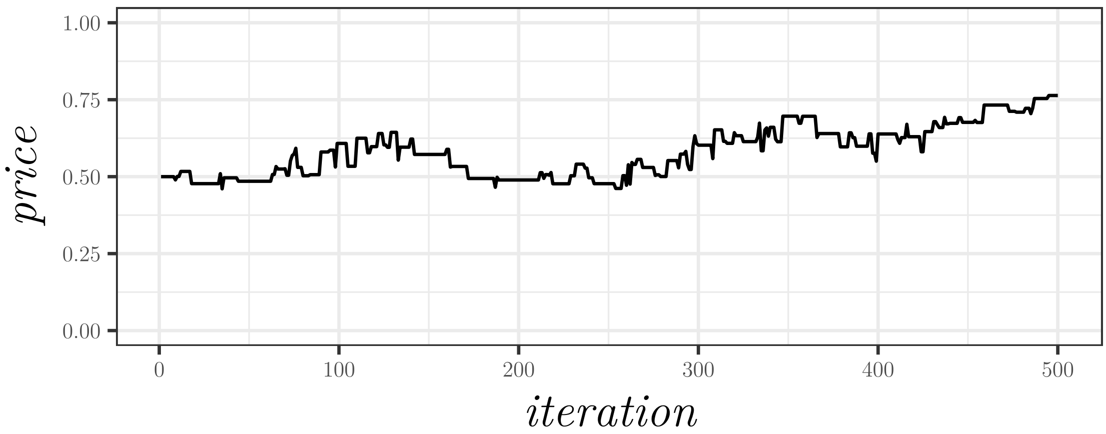



Figure 4.6: Trial-and-error example $\sigma = \pm 0.05$

in the model due to its nature and configuration.

### 4.4.2 LR-I strategy

The LR-I algorithm was introduced in [57] for application in dynamic spectrum agile markets. The algorithm underwent slight modification process to support the three-stage game by considering the cumulative profit resulting from the interaction on both the retail and wholesale markets, respectively.

The operator has a finite price level in the range $[\kappa_{min}, \ldots, \kappa_{max}]$, where $\kappa_{min} = 0.1$ and $\kappa_{max} = 1$, respectively. The remaining possible prices are uniformly distributed within this region. An action space of an operator is thus defined to be a probability vector $\mathbf{p} = [p_1, p_2, \ldots, p_M]$, where the operator selects the price $\kappa_j$ with the probability of $p_j$ and $M$ denotes the number of candidate price levels. The algorithm operates as follows:

1. The initial probability vector is defined as $\mathbf{p}(0)$.

2. At each time instant $t$, the operator chooses the price $\kappa$ based on the action probability vector $\mathbf{p}$. Thus, the operator chooses an action $a$ at instant $t$ based on the probability distribution $\mathbf{p}(t)$.

3. The operator receives the profit $\Pi_a(t)$ for the given action $a$.

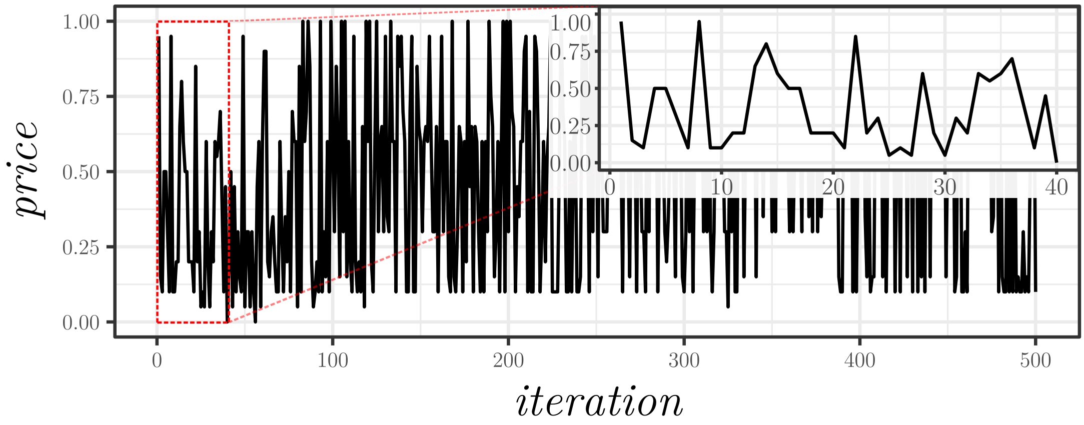



Figure 4.7: Linear-reward example $\eta = 0.04$

4. Each operator updates the action probability vector following the rule:

$$\begin{aligned} p_j(t+1) &= p_j(t) - \eta \Pi_a(t) p_j(t) \quad a(t) \neq p_j, \\ p_j(t+1) &= p_j(t) + \eta \Pi_a(t) \sum_{s \neq j} p_s(t) \quad a(t) = p_j, \\ j &= 1, \ldots, M, \end{aligned} \tag{4.8}$$

where $\eta$ is the learning parameter ($\eta \in [0, 1]$).

5. The algorithm stops when there are no incremental changes in the probability vector $\mathbf{p}$ between iterations.

In a price dynamic game with $K$ players ($K = 5$ in case of described model), provided each player (i.e., operator) uses the LR-I strategy, it is guaranteed that the game converges to a Nash equilibrium under the assumption that the game has only strict Nash equilibrium in pure strategies [58]. In our model implementation, a uniform initialization of the probability distribution function was used because it best reflects the assumption that there is no prior information available about the preferences. In the simulation conditions, the equilibrium manifests itself as a narrowing of the fluctuations accompanied by a shadowing of the impact of the initial distribution. A complete discussion about the low relevance attributed to the initial uniformity can be found in the seminal work by Oomen and Christensen in [59].

Example of price evolution controlled by the LR-I mechanism can be seen in Fig. 4.7. The figure shows the mechanism's attempt to optimize price selection from the initial state when the probability of choosing each available price is the same. Compared to the previous mechanisms, price evolution controlled by LR-I appears to be the most volatile over first $500$ iterations.

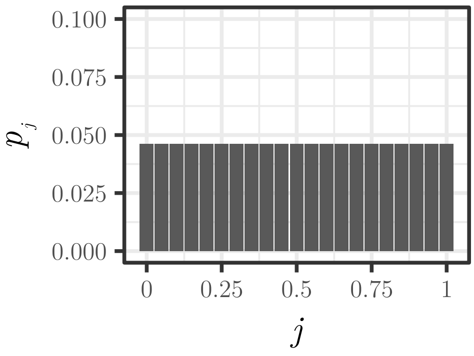
For further investigation of learning process plots showing price decisions, the evolution of probabilities affected by rewards, Tab. 4.4 can be used. Rewards in the example were random and uniformly distributed from interval $< 0, 1 >$ which is sufficient for mechanism demonstration. LR-I attempts to find the optimal price from interval $< 0, 1 >$ with price step $0.05$. Initial probabilities of choosing each price are shown in Tab. 4.4a). At the first iteration, price $0.95$ is chosen randomly earning the reward of $0.75$ which affects the distribution of probabilities according to equations 4.8 as shown in Tab. 4.4b). The probability of choosing price $0.95$ in the next iteration is increased significantly and therefore probabilities of other strategies are decreased slightly. However, price $0.15$ is chosen in the second iteration earning the slightly lower reward of $0.60$. Lower reward results in a lower increase of chosen price probability as shown in Tab. 4.4c) and also a lower decrease of other probabilities. In the following two iterations $0.10$ is chosen resulting in reward of $0.65$ Tab. 4.4d) and $0.50$ being rewarded by $0.40$. Probabilities of each strategy after four iterations are shown in Tab. 4.4e). Slight probability decrease of price $0.95$ chosen in the first iteration caused by following selections is clearly visible too. State of probabilities after 500 iterations is shown in a plot Tab. 4.4f) with modified $y$ axis interval. In contrast to the previously described Roth-Erev mechanism, multiple strategies scoring the lowest rewards have probabilities of being chosen equal to $0\%$ due to missing experimentation feature.

### 4.4.3 Successful-ratio strategy

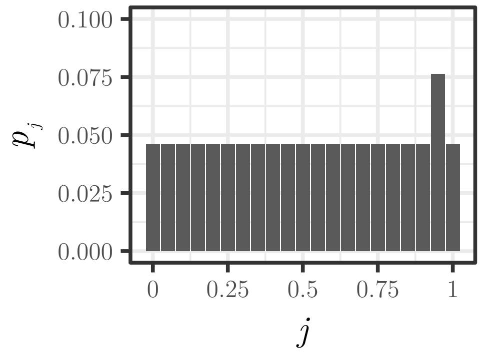
This equation was recently proposed in [41] and belongs to the family of adaptive pricing methods. Here, the price is dynamically adjusted in each time period, and the retail price $\kappa$ is adaptively accommodated as follows:

$$\kappa(t+1) = \kappa(t) + (\Psi(t) - 0.5) \cdot \mu, \tag{4.9}$$

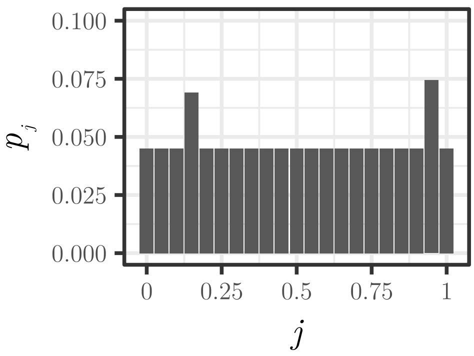
where $\kappa(t)$ is the channel price of the operator at time $t$, $\Psi(t)$ is the acceptance ratio of the operator ($\Psi(t) \in [0, 1]$), and parameter $\mu$ is the price change shaping parameter. In the price adaptation process, the evolution of the price is dependent



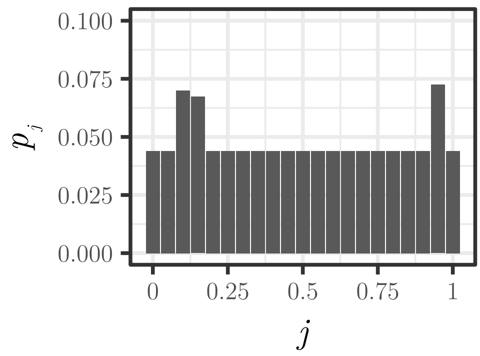

(a) Initial state



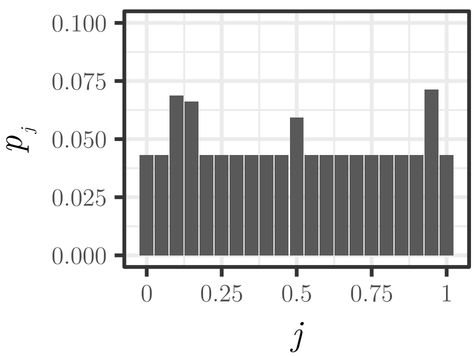

(b) After price 0.95 earns reward 0.75



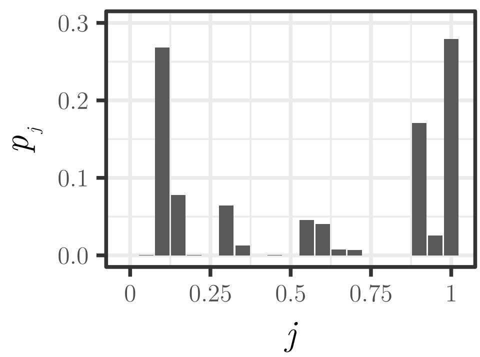

(c) After 0.15 earns 0.60




(d) After 0.10 earns 0.65




(e) After 0.50 earns 0.40




(f) Probabilities after 500 iterations

Table 4.4: Linear reward - Evolution of probabilities

on the average number of end-users that accept the offered price, as follows:

$$\Psi = \begin{cases} 1/2 & (BW_{avail} = 0) \wedge (S = 0) \\ 0 & (BW_{avail} > 0) \wedge (S = 0), \\ \frac{S^{idle->conn}}{S} & (BW_{avail} > 0) \wedge (S > 0) \end{cases} \tag{4.10}$$

where $S$ represents the number of end-users that maximize their acceptance probability $\mathcal{A}$ of connecting to the operator, $BW_{avail}$ is the number of unoccupied frequency channels of the operator, and $S^{idle->conn}$ is the number of end-users that accept the offer by connecting to the operator. Note that the acceptance decision of the agent has probabilistic characteristics that are determined by the acceptance probability of the end-users, and thus $S^{idle->conn} \leq S$. The pricing definition formulated in (4.9) ensures that operators establish the price in a given time frame based on their previous experience with end-user price acceptance and simultaneously, the definition offers smooth price evolution. Figure 4.8 illustrates the price adjustments performed by Successful-ratio mechanism according to valid randomly generated values of $BW_{avail}$, $S^{idle->conn}$ and $S$.

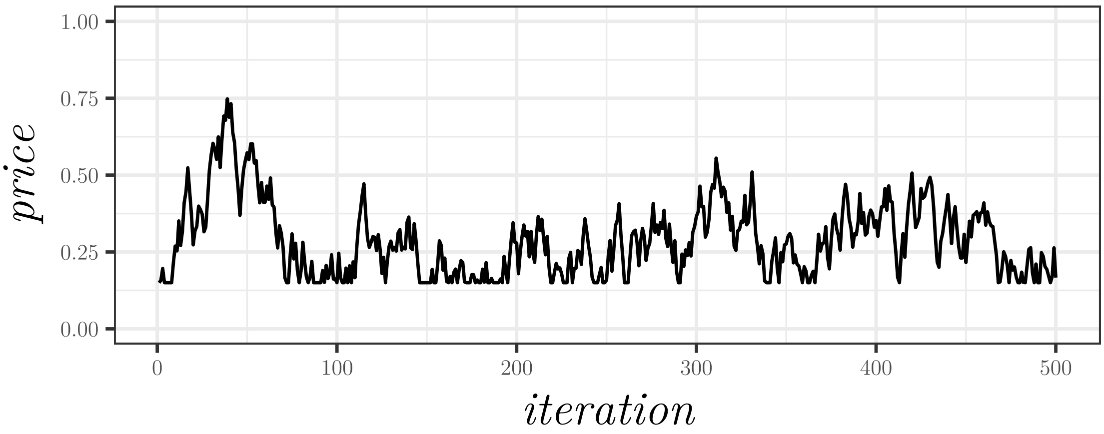



Figure 4.8: Successful-ratio example $\mu = 0.2$

In the applied behavioral models for dynamic regimes, there is a lack of access to real information about the future behavior of game opponents, so it is necessary to put more effort into analyzing either the strategies based on historical information (i.e., LR-I), or so-called myopic policy decisions (i.e., T-E and S-R). The time-filtered series, including information on the success or failures of previously applied strategies, can be an important guideline for future behavior. On the other hand, one specific class of suboptimal policies that has attracted a lot of attention is the class of myopic policies. In a myopic policy, we attempt to minimize the expected cost for each period within the period itself, while ignoring the potential effect on the cost in future periods. The myopic policy is attractive since it yields a base-stock policy that is easy to compute on-line, that is, it does not require information on the control policy in the future periods.

## 4.5 Investigated economic indicators

The role of an operator in the model is analogous to the role of investor who makes decisions about his portfolio with the aim to earn the profit, however, with a slight difference. There is no opportunity for operators to invest in multiple types of assets in the model since the whole frequency spectrum is treated equally despite the different characteristics of lower and higher frequencies in a real environment. The operators, therefore, take on the role of investors and invest their finances into a single-commodity portfolio (spectrum asset) with the expectations to earn the profit by attracting the demand from end-users using different pricing schemes. To examine and compare the performance of the investment opportunities in the model with the different configuration, a suitable measurement is required. Employed in Portfolio Investment Theory, Sharpe ratio is sufficient for the comparison and performance analysis.

Let $\Pi(t)$ denote the one-period profit of a spectrum asset between time $t-1$ and $t$ as:

$$\Pi(t) = \kappa(t)\Theta_r(t) - \Gamma(t)\Theta_w(t), \tag{4.11}$$

where $\Theta_w(t)$ and $\Theta_r(t)$ denote the spectrum asset purchased at time $t$ (i.e., number of channels) and the spectrum asset successfully contracted to retail end-users, respectively (i.e., $\Theta_r \leq \Theta_w$). The wholesale price $\Gamma(t)$ is provided by the SpecEx server and $\kappa(t)$ is the end-user retail price for the frequency channel set by the operator. In general, the profit of the operator is characterized by the revenue earned from the end-users using its services.

Now, let $\mu$ and $\sigma^2$ be the mean and variance of the profit:

$$\mu = E(\Pi(t)), \tag{4.12}$$

and

$$\sigma^2 = Var(\Pi(t)). \tag{4.13}$$

Stylized Sharpe ratio (SR) is then defined as the ratio of expected profit to the standard deviation of the profit:

$$SR = \frac{\mu - R_f}{\sigma}, \tag{4.14}$$

the expected profit is computed relative to the risk-free rate $R_f$ normally, a risk-free asset, such as short-dated government debt, earns the risk-free rate, [60]. For the purpose of investment analysis, the risk-free rate $R_f$ term is omitted as it is common for all scenarios. Moreover, in the standard definition of the Sharpe ratio, the returns on the investments are considered. However, as long as all investors are concerned with the same temporarily leased commodity, this can be replaced by the profits from the investments.

From Eq. 4.14 it is clear that any factor that affects the mean $\mu$ and variance $\sigma^2$ of the profit will also affect the Sharpe ratio of the spectrum investment. Thus, it is evident that the spectrum pricing strategy will directly influence the Sharpe ratio of the spectrum asset investment.

Instead of solely focusing on the profit, operators also need to be conscious of the risk and volatility, measured as the profit deviation, to which they are exposed. As risk-averse operators prefer high profits and low volatility, the alternative spectrum pricing method with the highest Sharpe ratio should be chosen when assessing investment possibilities. On the other hand, risk-seeking operators may take advantage of the spectrum pricing method that results in the highest risk (high profit deviation), but has the potential of earning high profits [61].

# 5 Agents' interactions and model configuration

Although a significant part of the agent-based model has been described in previous sections, it is necessary to provide a more detailed description of agent interactions to be able to implement the fully functioning system. Figure 5.1 is a graphical representation of the simulation process which consists of multiple iterations. Note, that few details including integration of S-R, T-E, LR-I, and end-users' state switching are not included in the diagram.

The simulation starts with the initialization of wholesale market by the SpecEx server. Depending on the desired number of iterations, stops, or continues by leasing of the frequency spectrum to operators, which consider the required volume according to the Roth-Erev mechanism. Leased channels are then offered on the retail market in multiple iterations. End-users evaluate each offer present in a system to determine the most convenient one, which is then accepted with the probability according to its relevance. In case of acceptance, a fee is being paid to the service provider, who maintains his allocated spectrum capacity during all retail market iterations nested in a single wholesale market iteration. Result of retail market iterations in time $t_0$ affects operator's behavior in $t_1$. The retail price is adjusted according to used pricing mechanism (S-R, T-E or LR-I) and also propensities of Roth-Erev's strategies are being updated. The process is repeated until the desired number of iterations is not reached.

Settings of the executed model have a significant impact on the results. The carefully chosen simulation parameters are given in Table 5.1. The parameters characterizing the behavior of the end-users ($\alpha$, $\beta$, $\gamma$, and $\delta$) were defined according to the recommendation given in [41] with the goal of simultaneously modeling them as quality and price sensitive. To demonstrate the robustness of the model in general, parameters related to the Roth-Erev algorithm ($e$ and $r$) were tweaked within the defined interval $\{0, 0.1, \ldots, 1\}$. The proposed sensitivity analysis of the Roth-Erev parameters provided the regions in the space where the investigated factors find their maximum and also show the transitions between these points.

Table 5.1: Table of parameter settings

| Notation | Value | Description |
|---|---|---|
| $L$ | $1,000$ m | length of the region |
| $P_s$ | $2N_0$ | signal power |
| - | $100$ | number of end-users |
| $\alpha$ | $0.05$ | price shaping parameter |
| $\beta$ | $0.8$ | price shaping parameter |
| $\gamma$ | $2$ | price sensitivity of the end-user |
| $\delta$ | $5$ | utility sensitivity of the end-user |
| $c$ | $4$ | acceptance probability coefficient |
| $\mu$ | $0.2$ | price-shaping parameter (S-R) |
| $\eta$ | $4 * 10^{-3}$ | price-learning parameter (LR-I) |
| $M$ | $20$ | price candidate levels (LR-I) |
| $e$ | $e \in \{0, \ldots, 1\}$ | *experimentation* parameter |
| $r$ | $r \in \{0, \ldots, 1\}$ | *forgetting* parameter |
| $N$ | $30$ | number of wholesale market strategies |
| $\Gamma$ | $0.2$ | wholesale price |
| $P_{act}$ | $1$ | activation probability |
| $P_{disc}$ | $1$ | disconnection probability |

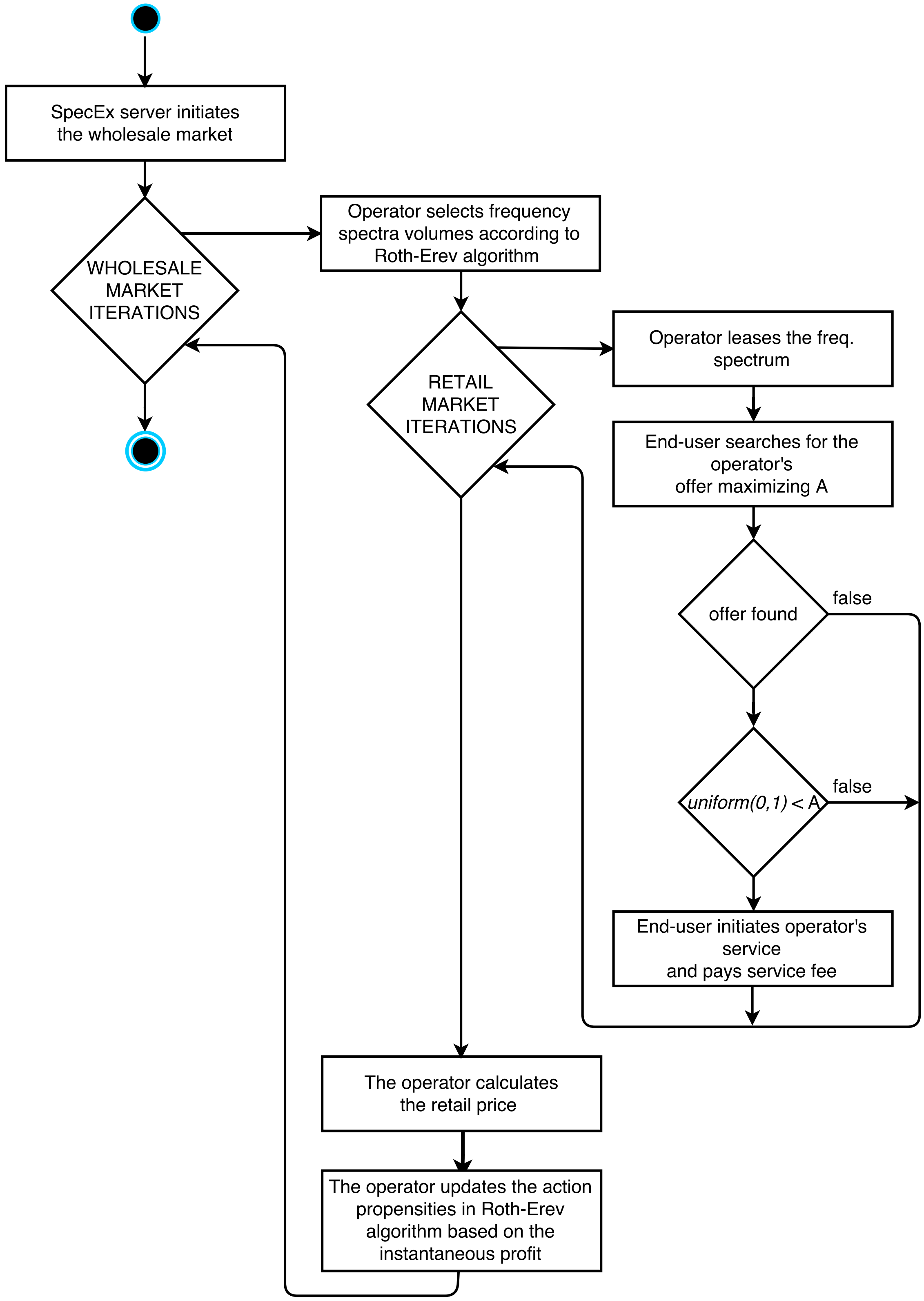



Figure 5.1: Diagram of interactions

# 6 Modeling tools and utilized software

Implementation environment determined by the objectives of a thesis is not considered to be generally recognized, therefore next two sections will be dedicated to the basic introduction with the modeling environment and a platform developed for the simulation purposes.

## 6.1 NetLogo

NetLogo [62] is the programmable multi-agent environment running in the Java Virtual Machine, well suited for creating complex evolving models [63]. According to U. Wilensky, one of its creators, it is an appropriate tool for both, the education and research, therefore, it is suitable for users with a lack of programming experience too. The tool is shipped with a large model library covering different areas of science including the documentation and the source codes, which may be helpful for the new programmers.

The language itself is not fully object-oriented. Classes called "breeds" are supported but inheritance as known from Java is not possible. Breeds extend the superclass called "turtle" but inheritance itself includes only attributes since methods are not tightly coupled with the classes. Further inheritance is however not allowed.

Three different types of agents are supported. Already mentioned turtles are first of them with the customizable shapes, color, attributes behavior, location etc. Further diversification is possible thanks to breeds. Different breeds may have different graphical interpretations and attributes. These are commonly used in more complex models, where multiple types of entities are required. Next, the tiles arranged in the matrix represent the environment. Tiles are programmable too, their shape and location is fixed. And lastly, a so-called "observer" which

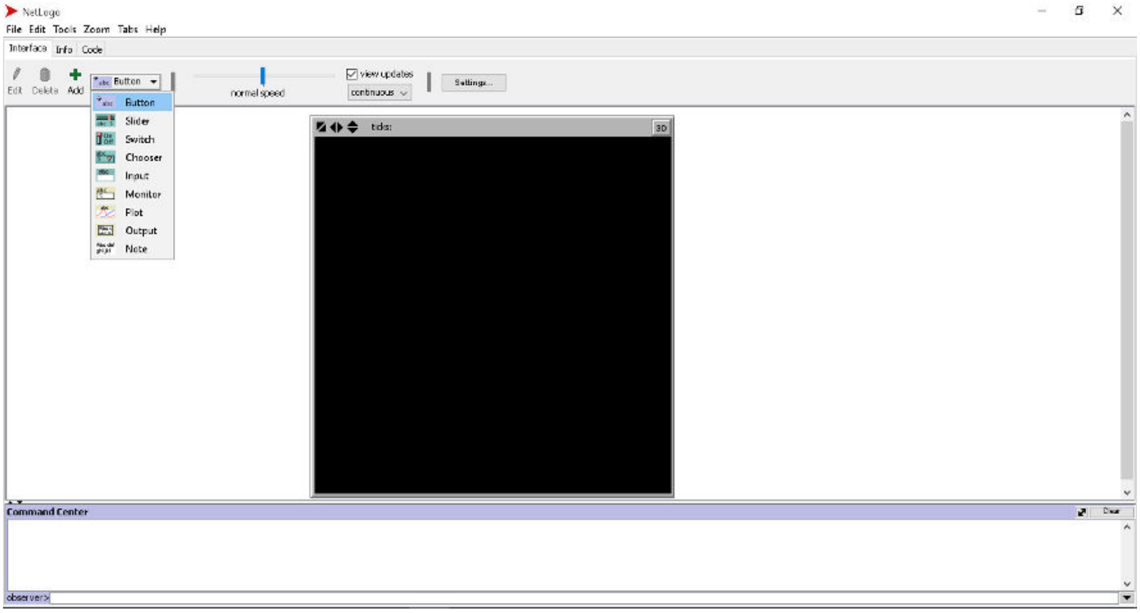


Figure 6.1: NetLogo GUI - Interface

mediates the communication between the programmer and the agents.

Another important aspect of simulation is the time. In the NetLogo, it is represented via a simple variable called "tick". From the technical point of view, it is the integer variable which is incremented on the call of "tick" method, its value can not be modified by any other way. When the value of the simulation time is changed, areas of the plots are updated as well as the simulation clock which is displayed during the simulation.

The graphical interface is user-friendly and intuitive. It consists of three cards: "Interface", "Info" and "Code". Large area for user-defined GUI of the model is the most dominant part of the "Interface". GUI creation is easy thanks to drag-and-drop principle. Numerous control components are available: button, slider, chooser, input, variable monitor, plot, output, note etc. Coupling with the code is intuitive, in the most cases, structures with the same name are tied together. Plots functionality may not be sufficient. Although single plot can visualize multiple variables at the same time using multiple pens with a legend and description added, it displays the axis numbering only on mouse hover. On the smaller screens, problems with a lack of space may arise when multiple plots are needed.

Documentation of the model is important, especially when models are shared. Programmers can use the simple documentation editor hidden under the card

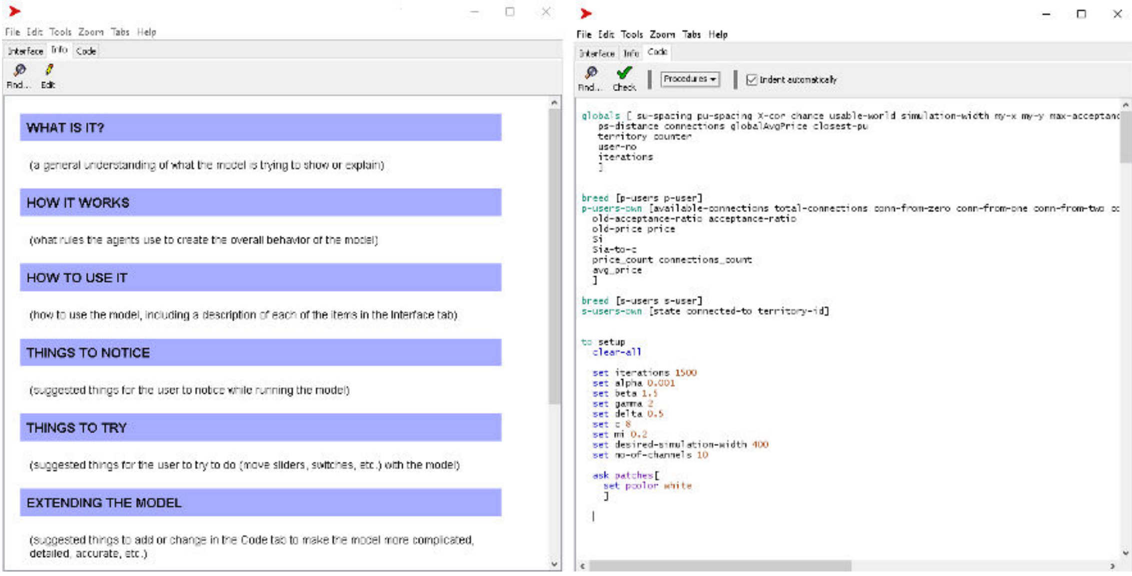



Figure 6.2: NetLogo GUI - Info and Code

labeled "Info". Preset structure makes writing and reading easier, moreover, it is a part of the model file so no extra effort is needed for distribution with the implemented model.

Code editor, the "Code" card, lacks some useful features compared to well-known integrated development environments. On the other hand, each new update introduces new features that make writing a code more pleasant experience. Latest update, version 6.0.2, brought functions like: possibility to move a code, jump to declaration/usage, lines numbering etc. A vast majority of included models are written in the single file, but it is also possible to split code into multiple files and use them thanks to the "include" functionality, however, this approach requires more planning to gain an advantage over standard single-file approach. As was already mentioned, NetLogo does not fulfill OOP and therefore language itself does not force writers, for example, to code a breed functionality into its corresponding file as known form Java. Support from the editors part could be improved too.

BehaviorSpace is a useful feature when a model's behavior needs to be verified with different setting combinations. Figure 6.3 shows an example of Behavior space configuration to test simple NetLogo model of Fire [64]. Using this tool, all desired variables can be varied over specified intervals or given specific values with the possibility to repeat each combination multiple times to obtain more

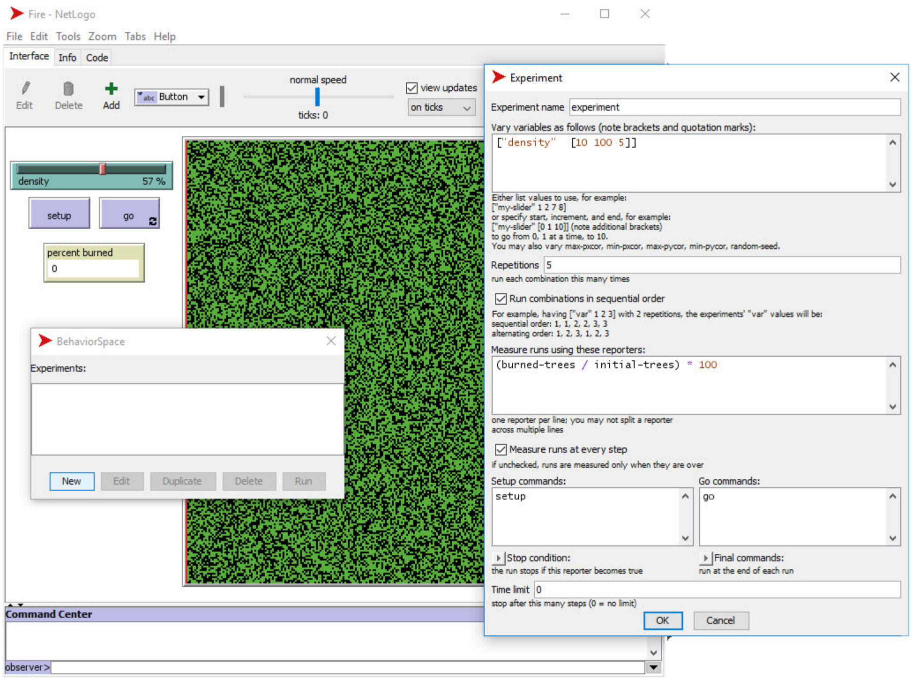



Figure 6.3: NetLogo - BehaviorSpace

results for later aggregation. To perform the experiment the following parameters needs to be defined: allowed values of varied variables, number of repetitions each combination will be executed, reporters of desired output attributes, so-called setup and go commands and optional stop conditions, final commands and time limit. Upon defining everything necessary, the experiment can be executed with the combinations running in parallel. The execution time is, of course, dependent on the hardware, especially central processing unit and the amount of random-access memory (to avoid swapping on the hard drive when running memory demanding models).

One of the greatest features of the NetLogo is the presence of Java API. Models can be executed and controlled from the external code written in Java, Scala, R etc. Performing loops from the external application may improve the performance in some cases [65] too, but what one may finds really handy is the possibility to write the scripts that can execute the multiple simulations in parallel, gather and process the data after simulations are finished and plot the results with a single click.

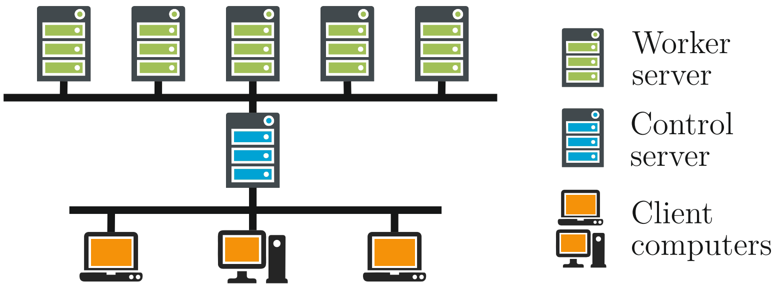



Figure 6.4: NetLogo Launcher - Architecture

## 6.2 NetLogo Launcher

The functionality of already described BehaviorSpace may be found insufficient for demanding experiments because the execution of simulations is limited to a single computer. When a large number of parameter combinations is required, a powerful computer is needed that is able to serve the results in acceptable times. Hoverer, in case such hardware is not reachable a different approach may be taken which consists of utilization of multiple computers at the same time to run the simulations in parallel across all available machines.

Figure 6.4 illustrates the architecture of custom-made solution [66] that makes carrying an experiment across multiple computers possible. In case of the implemented solution, we call them Worker servers. These servers may be regular workstations, servers or notebooks running Java application. However, notebooks are not recommended for this purpose due to, for such experiments common, high long-term system load that may damage the internal components. Worker servers capable of running NetLogo simulations are not contacted by users directly. A device acting like a bridge between users and Worker servers responsible for distribution of simulation requests is called Control server. It is not only responsible for the distribution of requests but also for monitoring of running simulations, gathering of results and monitoring of worker servers' processor load and memory utilization. After termination of the experiment, all obtained results are being sent to an e-mail address specified by the user upon simulation request submission.

Both Worker and Control servers are capable of handling multiple requests incoming from same or different users. Each user of the system has its own gen-

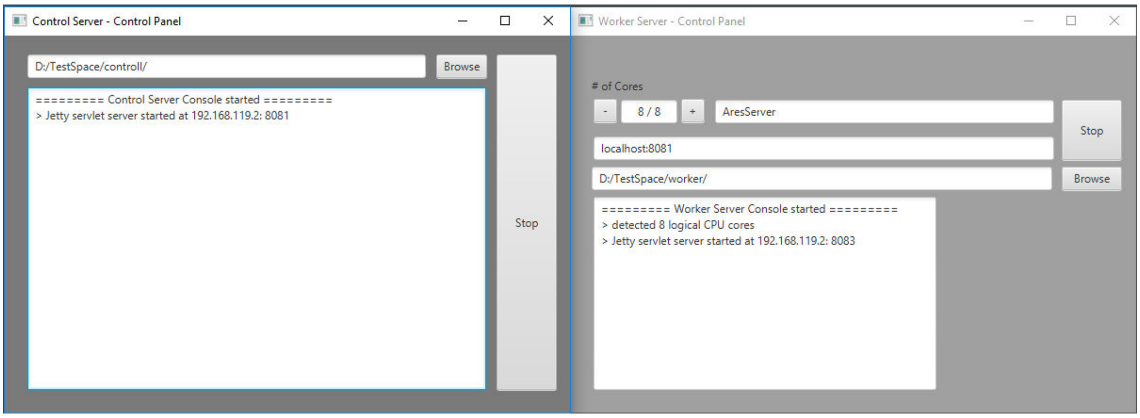



Figure 6.5: NetLogo Launcher - Server GUI (left Control, right Worker)

erated unique token identifier which is used for the authentication and the authorization. In case multiple requests were submitted, these are scheduled in the way that no request will suffer from so-called starvation that may negatively impact user experience. The progress of users' active request can be tracked using a featured list that also contains an estimation of finish time that is calculated from already processed simulation instances.

The user interface of server part is rather simple as shown in Fig. 6.5. The only required parameter of Control server is the path where incoming requests and results will be stored for each conducted experiment. On the other hand, Worker server calls for more complex yet still not too complicated configuration. It is possible to run both Control and Worker server on the same machine. The path where local requests and results will be stored is required too. When needed, it is possible to limit the number of processor cores that will be utilized by Worker by parallel execution of configured simulation instances. Address of Control server and also the local server name needs to be filled in order to successfully connect Worker server with the Control Server. Figure 6.5 shows both server applications up and running on the same computer listening to different ports.

GUI of a client application is shown in Fig. 6.6. It offers functionalities similar to the official BehaviorSpace. The figure shows an execution of the same model as shown in Fig. 6.3 with the same configuration. Submitting new simulation request requires specification of the following details: pathname of NetLogo model file or a .zip archive containing needed files (feature implemented for the multiple-file NetLogo models), starting commands, desired values or intervals of

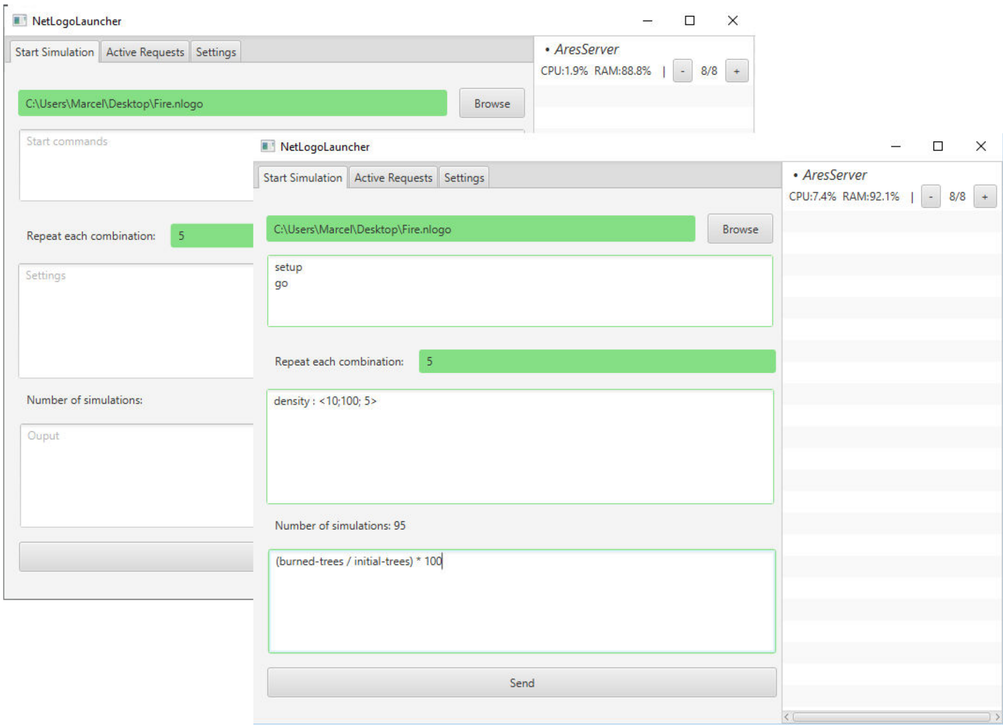



Figure 6.6: NetLogo Launcher - Client GUI - New request

varied variables, number of repetitions and finally output variables or NetLogo reporters. The right part of the GUI contains the list of available Worker servers with their current state displayed. The user is able to choose how many cores of each server he wants to utilize. Simulation instances will be then split according to the defined number of cores.

Each part of implemented software solution was written in Java due to its portability that allows easy deployment on any computer running Java Virtual Machine. Simple graphical interfaces were designed in JavaFX and the RESTful API capabilities of both Worker and Control server were implemented using Jetty. Correct functionality is being tested by JUnit test cases during the process of development. Further plans for software evolution exist since the project is in its early stage of a lifetime. These plans include improvement of security features, stability improvements, recovery capabilities and overall improvement of user experience.

# 7 Results and evaluation

Figure 7.1 shows the characteristics of retail pricing mechanisms used by the Operator III in a simulated model with the following configuration: $e = 0 \wedge r = 0$, i.e., *cumulative payload matching*. In the Fig. 7.1a) evolution of retail price is shown. While both the S-R and T-E strategies vary around a certain mean $\mu$ with a variance $\sigma^2$, the LR-I algorithm converges to a ß, stable equilibrium significantly lower than $\mu$ of S-R and T-E. This is caused by the end-users' price sensitivity and the properties of the LR-I algorithm. Coupling of these two attributes results in an emergent phenomenon called price-war which forces the prices to drop. This results in the minimum acceptable return that assures the operators of a positive profit. The existence of the price-war in the model confirms the validity of the model, as per other recent works (e.g., [57]). Effect of price-war is also visible in a Fig. 7.1b), which shows the average price on the retail market. An initial phase of a price war, the gradual decline of retail prices, is also clearly visible in the Fig. 7.2. Operators push down the retail prices to attract the price-sensitive end-users until the point where there is no further possibility of lowering the price due to expenses being equal or higher than actual return. The LR-I algorithm has the lowest average retail price followed by those of the S-R and T-E algorithms. Geographical location of the operators has no significant impact on the retail prices.

Figure 7.1c) illustrates the average profit of the operators. The highest profit is achieved by the S-R strategy followed by the T-E which tends to maintain higher retail prices, therefore, discouraging end-users. On the other hand, the price-war that occurs when the LR-I strategy is used achieves the lowest yet still relatively high profit when its mean retail price that attracts the highest number of end-users is considered. The relevance of a central spatial location (in terms of end-user demand attraction, i.e., increased profit) indicates the emergence of the real phenomena [67], which confirms the validity of the model.

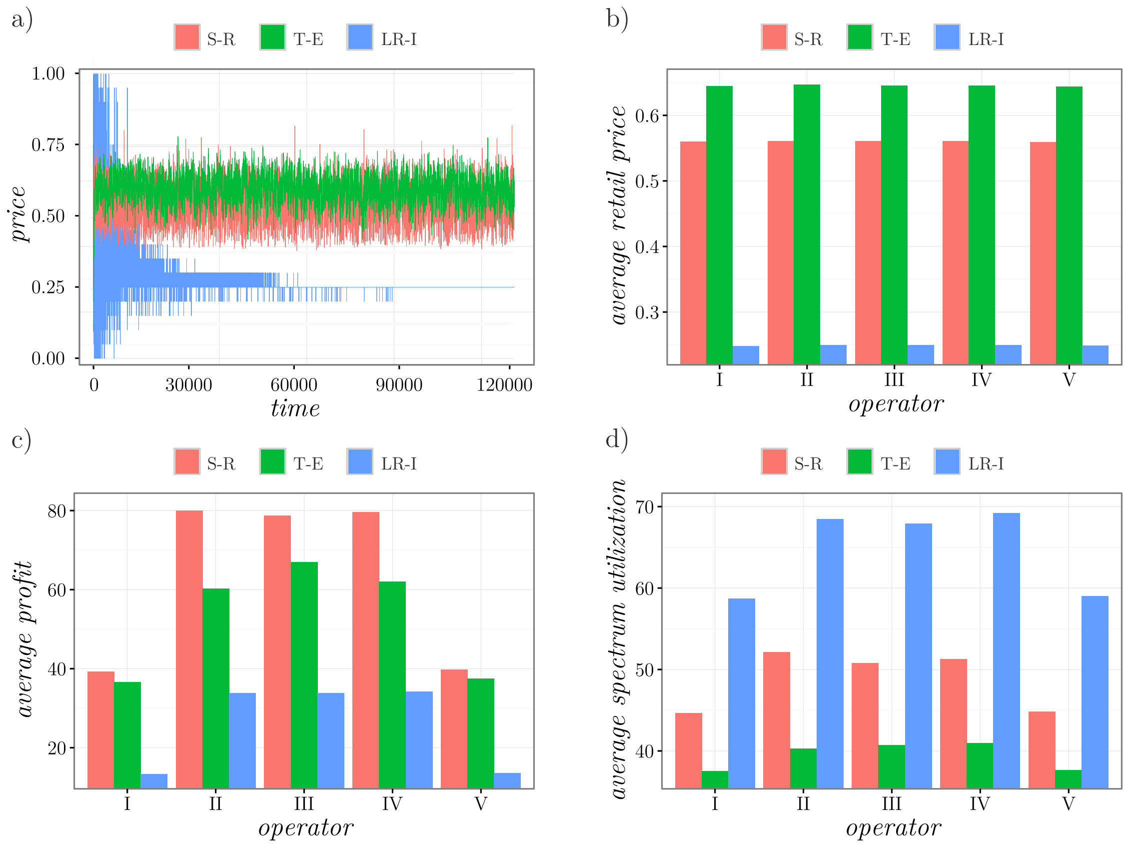



Figure 7.1: Evaluation of retail price with tested methods

Figure 7.1d) shows the average spectrum utilization ($E(\frac{\Theta_r}{\Theta_w})$) under the given conditions. It is evident that the end-users benefited from the price-war that occurred in the market when the LR-I strategy was used. This showed that they are usually willing to accept low prices, which in turn results in the highest spectrum utilization.

The emerging phenomena observed within the simulations indicate the validity of the model with respect to the previous works i.e., price war [57], central spatial location dominance [67], and end-user price sensitivity [68], therefore, we believe that the proposed agent-based model accurately reflects operations on the real-time spectrum trading market.

An understanding of the interactions among various strategies can be extremely valuable for the operators who wish to ensure economic efficiency and stability. The Nash equilibrium could provide a theoretically satisfactory framework in the cases where multi-agent systems operate close to the static equilibrium. However, the dynamic properties of the analyzed model are often of equal or greater

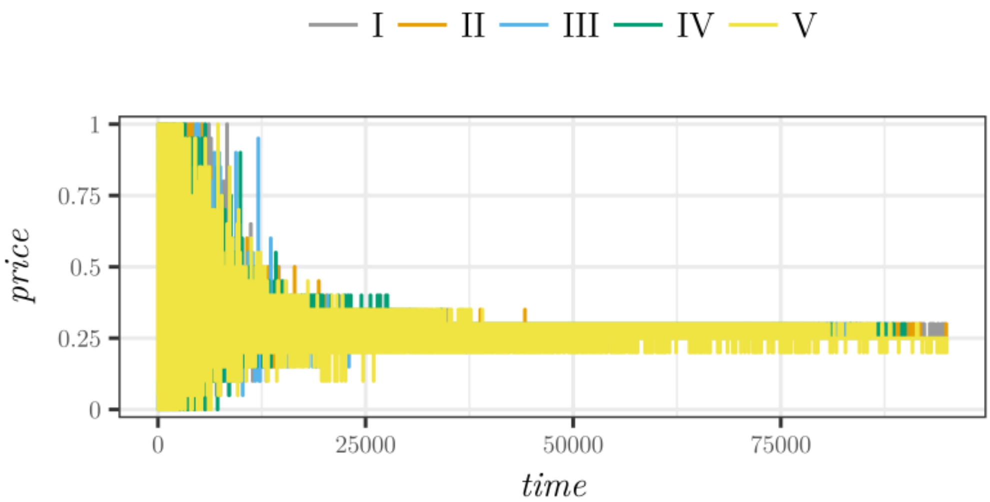



Figure 7.2: Price-war on retail market (wholesale price = $0.1$)

concern. Even in a real environment, not all DSA agents have the knowledge and computational resources to compute the equilibrium. Even when the agents have this common knowledge and the resources, it is still desired to address the question of which equilibrium is chosen and how the agents coordinate to reach it. Thus, the basic terminology of game theory can be used, including the well-known concepts of *pure strategy* and *mixed strategy* into the discussion [69]. The concept of pure strategy refers to the complete definition of how the agent will play the game in different elementary situations dictated by the environment. In contrast, more complex behavior and decision-making can be obtained by a hierarchical and probabilistic mixture of several pure strategies, i.e., the concept of mixed strategy.

Figure 7.3 shows the Roth-Erev action probabilities $p_i(t)$ of the Operator I and Operator III with the parameters $e$ and $r$ chosen to illustrate possible scenarios discovered by sensitivity analysis over parameter space. Interestingly, found concepts are often discussed in the traditional game theory. In other words, the decisions of the operators on the wholesale market create four fundamentally different regimes. Based on the results of extensive simulations, the existence of these regimes is not coupled with any specific retail pricing scheme and, for illustrative purposes, the results from the simulations with S-R applied are used.

Parameter set $e = o \wedge r = 0$, which can be also described as *cumulative payoff* *matching* learning, converges to the stable *mixed strategy* profile for both Operators I and III as can be seen in the Fig. 7.3a)-b). However, the mixed strategy equilibrium is not asymptotically stable as *cumulative payoff matching learning* suffers from the *habit forming* [52]. This means that the decisions of the operators are concentrated on certain actions simply because they were taken early and often. It should be noted that Operator III, which is located in the geometric center of the investigated region, leases a higher amount of frequency spectrum than leased by Operator I on average. The existence of the *mixed strategy* profile in the decision process of the operators in the micro-economy level signalizes the presence of the *strategic obfuscation*. As will be shown later, the strategic obfuscation may in certain cases decrease the profit of the obfuscating operator compared to the situation when *pure-strategy* is present in the decision process of the operator.

Figures 7.3c) and d) illustrate the situation with parameters $e = 0 \wedge r = 0.1$. This set of parameters is very important because in this case, the operators' probability vector collapses into the case characterized as a pure strategy ($i$ exists, where $p_i = 1$). As can be seen, the deterministic strategy remained stable over the investigated time interval and Operator III systematically leased a larger amount of frequency spectrum than its competitor. Holding on to the chosen strategy is natural and common in the micro-economy if it warrants success and results in profit [70].

Another possible scenario can be illustrated with the parameter set $e = 0 \wedge r = 0.1$. Figure 7.3e) shows so-called *win-stay, lose-switch*, which is often observed in the real conditions when are social or economic entities exposed to a decline in utility or profit exhibit preferences for switching instead of stagnation. It makes the operators switch the action if the payoff is below their aspiration level, otherwise, the strategy is repeated. Finally, Fig. 7.3f) shows the situation with the parameter setup $e = 1 \wedge r = 1$. This example can be considered as a case of the mixed strategy profile where ($\forall i, i \in N, p_i$ is equal), which can be interpreted as the *random selection* strategy.

The results illustrate how a wholesale market modeled by Roth-Erev algorithm behaves under different sets of guiding parameters. Note that only the bordering set of parameters are included in the simulations $e \in \{0, 1\}, r \in \{0, 0.1, 1\}$ and the configuration of the remaining parameters governs the transitions among these regimes.

As was stated in one of the previous sections, Sharpe ratio is a measurement

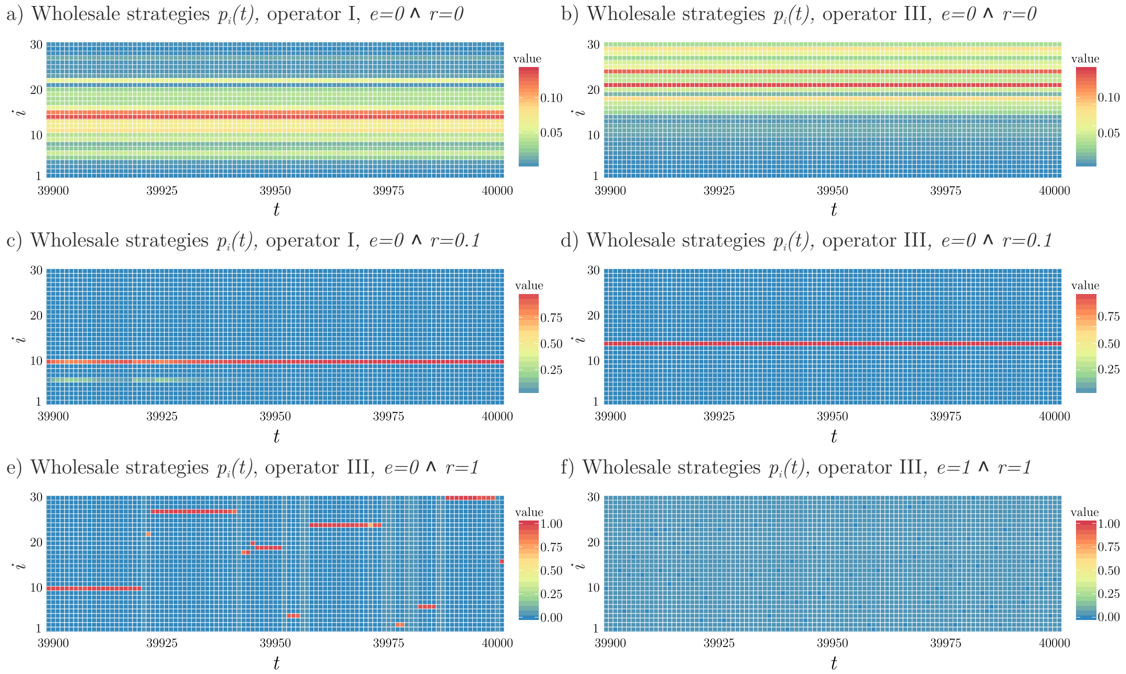



Figure 7.3: The dynamics of wholesale market regimes. The color levels are proportional to the probabilities $p_i(t)$ of the strategies $i = \{1, 2, \ldots, N\}$ at given time $t$ (index $i$ refers to the number of contracted frequency channels)

method suitable for financial performance analysis and comparison of investment opportunities, therefore, it is a suitable way to determine whether the tested pricing strategies suit the different types of trading operators. The operators, which take on the role of investors, can be risk-averse with respect to this uncertainty they may consider both the average profit $u$ at time $t$ and the volatility of that profit $\sigma^2$. In contrast, risk-seeking investors are willing to accept greater volatility and uncertainty in their investments in exchange for anticipated higher profits. A Sharpe ratio greater than 1 is rated acceptable by risk-averse investors and a ratio higher than 3 is rated as very good [71]. Nevertheless, risk-seeking investors by virtue of their nature seek to maximize their long-term profit with no regard for the perceived profit volatility. To determine the suitability of used pricing methods, extensive parameter sensitivity of experimentation parameter $e \in [0, 1]$ and recency parameter $r \in [0, 1]$ was performed (Fig. 7.4).

The parameters $e, r$ affect the Sharpe ratio only marginally in case of T-E strategy (Fig. 7.4a). The values of the Sharpe ratio are relatively low $[0.6, 0.8]$ which suggests that the T-E strategy is not a promising investment opportunity for the operators. The corresponding average profits as a function of the $e, r$ parameters are plotted in Fig. 7.4d).

On the other hand, the LR-I strategy provides interesting results with its Sharpe ratio values from interval $[1.4, 5.8]$(Fig. 7.4b)) with a maximum reached for the set of parameters r$\in \{0.1, 0.2\} \wedge e \in \{0\}$. The operators are guaranteed a very stable profit with low volatility when they operate in the regime characterized by the existence of pure strategy in their decisions (Fig. 7.3c)). The *Strategic obfuscation* regime ($e = 0 \wedge r = 0$) provides rather worse results, followed by the *win-stay, lose-switch* ($e = 0 \wedge r = 1$) regime. As the results indicate, the LR-I strategy is perfect for use by risk-averse operators willing to maximize their Sharpe ratios.

The S-R pricing strategy provides Sharpe ratios in the range of $[1.2, 2]$ (Fig. 7.4c)). As in the previous case, the Sharpe ratio reaches its maximal values in the parameter interval $r \in \{0.1, 0.2\} \wedge e \in \{0\}$. Although the Sharpe ratio of S-R is notably lower compared to the LR-I strategy, it provides larger profits across the whole parameter space than those of the corresponding LR-I strategy as can be seen by comparing Figs. 7.4e) and 7.4f). The Sharpe-ratio is lower in this case at the expense of higher profit volatility. The simulation results suggest that the S-R strategy serves as a worthwhile pricing strategy for operators with risk-seeking characteristics.

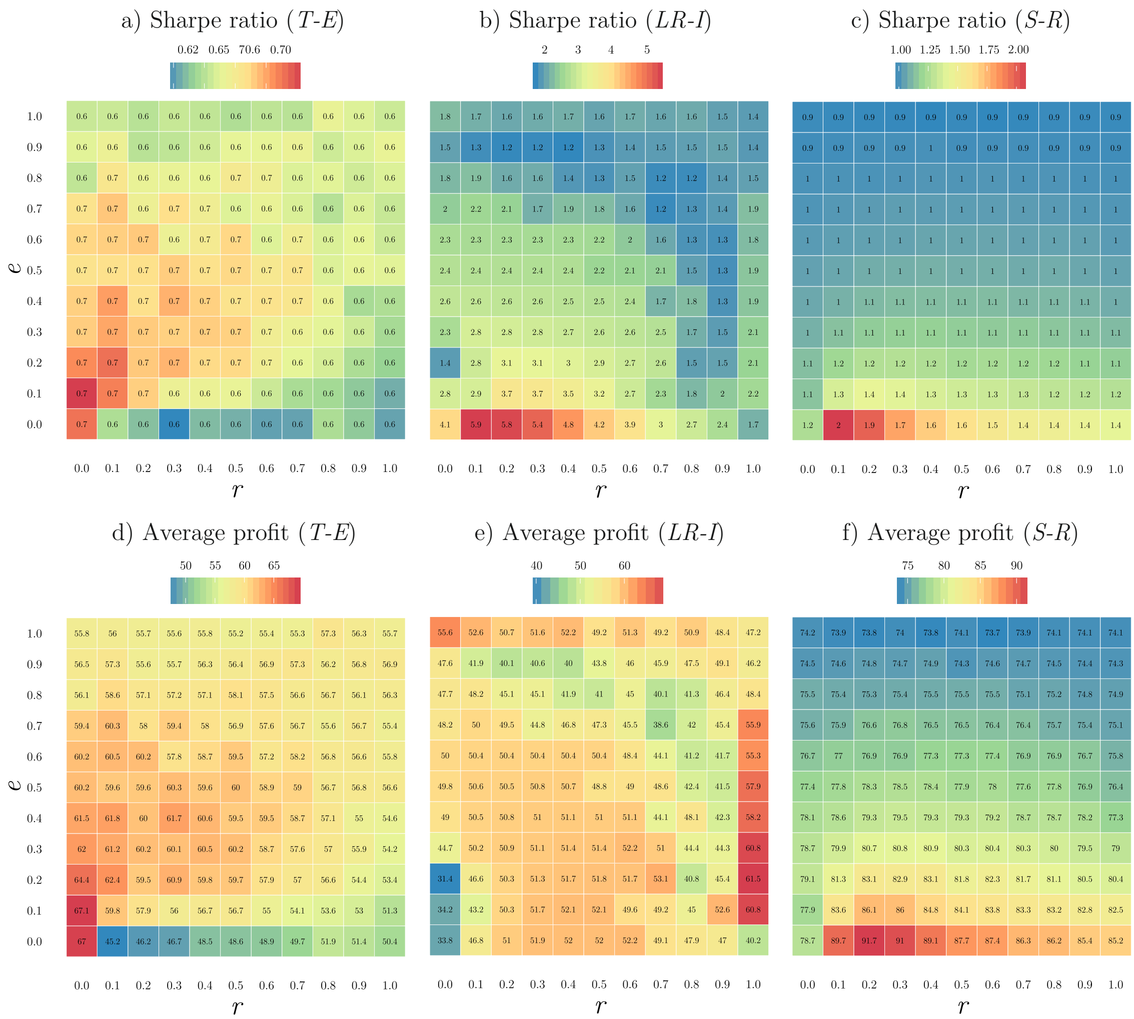



Figure 7.4: Heat maps of sensitivity analysis related to the $e$ and $r$ parameters, the Sharpe ratio and average profit

# 8 Conclusions

Frequency spectrum became the scarce resource due to the mass utilization of wireless technologies. The gradual increase of a data traffic and the expected breakthrough in numbers of connected devices caused by the arrival of 5G in 2020 requires replacement of the outdated regulatory models that are not flexible enough. Moreover, using only licensed spectrum may not be sufficient soon, therefore one of the key features of dynamic spectrum access networks, secondary spectrum usage, will be needed to meet requirements of 5G and satisfy the growing demand. License holders are allowed to lease unused spectrum to the secondary users and so prevent the emergence of spectrum holes which improves the spectrum utilization in DSA network and provides a reasonable profit to the primary license holders.

The issue of pricing on the secondary spectrum market, investigation of which was the main objective of this thesis, is a widely studied topic with the raising importance. To study nature of open access network that utilizes various pricing mechanisms an agent based model of the wholesale and retail market spectrum distribution was implemented in the NetLogo simulation environment.

On the modeled wholesale market, learning mechanism called Roth-Erev algorithm was used to control the amount of leased frequency spectrum. Four different regimes of the operators' behavior were analyzed *strategic obfuscation*, existence of a stable *pure strategy* in the decision of the operator, *win-stay, lose-switch* and *random selection*. On the retail market, three different pricing strategies were utilized namely, LR-I, S-R, and T-E. Best results were achieved by more advanced schemes LR-I and S-R in the regions where the operator follows the *pure strategy* in his decisions on the wholesale market (parameter set $r \in \{0.1, 0.2\} \wedge e \in \{0\}$), which showed that *forgetting* parameter has the potential to increase the performance of the measured indicators. Nevertheless, the *experimentation* parameter does not bring any relevant improvement. The results of the sensitivity analysis focused on the parameters of the Roth-Erev algorithm demonstrated broad consistency with the aforementioned regimes.

For the financial performance analysis and comparison of investment opportunities in the simulations with different pricing mechanisms, Sharpe ratio measure was used which showed that the LR-I pricing strategy, achieving Sharpe ratio values from $[1.4, 5.8]$, can be used to advantage by operators who are characterized as risk-averse investors. On the other hand, S-R with lower Sharpe ratios ($[1.2, 2]$) and a promise of higher profit is convenient for risk-seeking investors.

The most significant finding to emerge from this research is that the spectrum pricing strategies determine the characteristics of the operators, who in this case take on the role of investors. Moreover, the study also revealed that certain discounting of the past profit values in the system has the potential to increase the profit for operators, provided that a more advanced pricing scheme is used (e.g., LR-I or S-R).

Although the proposed agent-based model of spectrum trading in the simulated open-access network served well for the purpose of wholesale and retail market analysis and the analysis of the nature of operators' behavior, we are aware of its limitations. These include the overly simplified topology of the network with the heterogeneous BTSs and end-users as well as the omission of the real-world environment influences. To avoid the presence of interference, simulated BTSs utilized different frequencies, on the other hand, slightly different characteristics of these frequencies were not taken into account but we believe this did not void the conclusions made.

The extensive topic of wireless communication, cognitive networks, and dynamic spectrum trading opens space for further research. Introduction of base stations with various characteristics, e.g. macrocells, static femtocells, dynamic femtocell mounted drones [72] etc., coupled with sophisticated traffic splitting and routing mechanisms will improve the relevance of model significantly as well as the extension of the current model with the heterogeneous agents and 3D environment can result in interesting results with the opportunity of comparison with the already gathered data. Mobility models can be used to overcome the limitations caused by static end-users and the future work may benefit from the real data instead of relying on the randomly generated end-users' demand.

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# Appendixes

**Appendix A** CD with the thesis and source codes of NetLogo model