University of Economics, Prague

Faculty of Informatics and Statistics

Department of Information Technologies

Study programme: Applied Informatics

Field of study: Information Technologies

Optimizing Customer Lifetime Value in Retail Banking

MASTER THESIS

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2018

Declaration

I hereby declare that I am the sole author of the thesis entitled "Optimizing Customer Lifetime Value in Retail Banking". I duly marked out all quotations. The used literature and sources are stated in the attached list of references.

In Prague on April 24, 2018 ....

Tomas Pihrt

Acknowledgement

I hereby wish to express my appreciation and gratitude for help and support to the supervisor of my thesis, Ing. Pavel Jasek. Furthermore my family, mainly parents who encouraged me during my whole studies.

Abstract

Each year banking institutions spend billions of dollars on marketing incentives to stay competitive. Whilst focusing on short-term metrics e.g. profit or revenue, they promote products and services which do not completely fit customer's needs. To measure the impact of marketing campaigns on customer loyalty and future behavior, a customer-centric model based on Customer Lifetime Value in combination with a Markov chain model is suggested. Treating customers as company's assets is an essential assumption for long-term profitability and growth. To optimize the marketing resource allocation, it is crucial to cover not only one step marketing campaigns (e.g. next best offer), but the overall marketing strategy, bearing in mind all possible consequences. Such an approach can be compared to strategic games and is best described by a Markov Decision Process. The proposed method, which covers the whole retail banking marketing process from the campaign definition up to the result optimization, can be regarded as the main contribution of this thesis. The comparison of existing and CLV optimal marketing strategies of a mid-size European bank is provided to validate the modeling approach on a dataset containing more than 5 million customers.

Keywords

Customer Lifetime Value, Markov Chain, Markov Decision Process, Retail Banking, Marketing Campaigns

Abstrakt

Bankovní společnosti po celém světě utratí každoročně miliardy dolarů za marketingové výdaje, aby udržely své postavení v konkurenčním prostředí. Zatímco se tyto firmy soustředí na krátkodobě orientované metriky (např. obrat či zisk), nabízí zákazníkům produkty a služby, které plně neodrážejí jejich potřeby. Zákaznicky orientovaný model založený na principu Customer Lifetime Value v kombinaci s Markovským řetězcem je navrhován, aby bylo možné měřit vliv marketingových kampaní na spokojenost zákazníků a jejich budoucí chování. Změna ve vnímání zákazníka jakožto aktiva podniku je základním předpokladem pro dlouhodobou ziskovost a růst. K nejvhodnějšímu rozvrhnutí marketingových zdrojů je zapotřebí se zaměřit nejen na optimální výběr následující marketingové kampaně, ale vyhodnotit také vliv celé marketingové strategie se všemi jejími důsledky. Takový přístup je srovnatelný se strategickými hrami a nejlépe jej lze modelovat pomocí Markovova rozhodovacího procesu. Navržená metoda, která pokrývá celý marketingový proces v retailovém bankovnictví od tvorby marketingové kampaně až po optimalizaci výstupu, je hlavním přínosem této práce. Srovnání stávající a optimální marketingové strategie (s ohledem na CLV) středně velké evropské banky s více než 5 miliony zákazníky je uvedeno za účelem ověření navrhovaného přístupu.

Klíčová slova

Customer Lifetime Value, Markovský řetězec, Markovův rozhodovací proces, retailové bankovnictví, marketingové kampaně

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Chapter 1

Introduction

Marketing campaigns are getting more and more complex as corporations strive to gain new customers or to keep existing ones. These conditions are caused mainly by new technical capabilities that have developed over the past decades. Technologies such as big data processing and machine learning offer tailored real-time customer interaction and experience. Recent research conducted by (Econsultancy, 2014) and (Earnix & Marketforce, 2017) has revealed how important and currently relevant this topic is.

Campaign management is a decisioning process that can be driven by diverse metrics e.g. customer satisfaction, customer loyalty and customer lifetime value. It can also be product driven, where product managers oversee optimal resource allocation (Kotler & Keller, 2011). Moreover, marketing campaigns can be divided into three main categories – acquisition, cross-selling and up-selling, and finally retention. To sustain profitable long-term growth, the marketing strategy must simultaneously manage customers throughout the whole customer life cycle. (Triplett, 2012)

Mathematical and statistical methods are used to determine the selected metric's change over time. Models such as the Recency-frequency-monetary (RFM), Markov Chains, Bayesian approaches (Pareto/Negative Binomial Distribution) and Regression were applied throughout various industry sectors. (Ekinci, Ulengin, & Uray, 2014)

The aim of this Master's thesis is to develop and implement an algorithm optimizing marketing campaigns in the retail banking industry. Key industry specifics are long-standing relationships with customers and detailed historic information about customer behavior. As implied by the name "retail banking" a customer is regarded to be a person who has an active relationship with the bank (e.g. open checking account, cash loan, insurance). Companies are therefore excluded from this study. Firstly, the key drivers of customer relationships and their dynamics must be identified, and secondly the appropriate modeling techniques which are able to learn from the rich data background are selected to provide an optimal solution.

The thesis is divided into chapters, structured as follows. Chapter 2 (Literature review) gives a general overview of the current marketing approaches and used optimization techniques across industries. Chapters 3 & 4 describe in detail the customer metrics and predictive modeling in the retail banking. Chapter 5 contains a description of the research method development and implementation. The model comparison and performance are mentioned in chapter 6. All findings are summed up and discussed in the final chapter 7. All chapters should be read sequentially since they are chronologically ordered with respect to the aim of the thesis.

Chapter 2

Literature review

The aim of this chapter is to evaluate previous research on the topic of marketing campaigns optimization. Technologies and methodologies used across diverse industries provide a solid background for further specialization at retail banking.

2.1 CLV surveys and researches on CLV utilization

Companies Econsultancy and Sitecore conducted a research (Econsultancy, 2014) on Customer Lifetime Value (CLV)[1] with the context of loyalty and revenue. In their research there were almost 900 respondents from various business sectors. The 4 business sectors represented by most of respondents were: Retail (22%), Technology, Media and Telecoms (19%), Financial Services (12%) and Travel and Leisure (7%). The research is mentioned here to demonstrate how non-technically oriented respondents perceive the importance of single customer view, customer experience and interaction between online and offline channels which is all driven by technology integration. Major findings are presented in the following figures.

[1]Customer Lifetime Value is defined in the chapter 3.2.

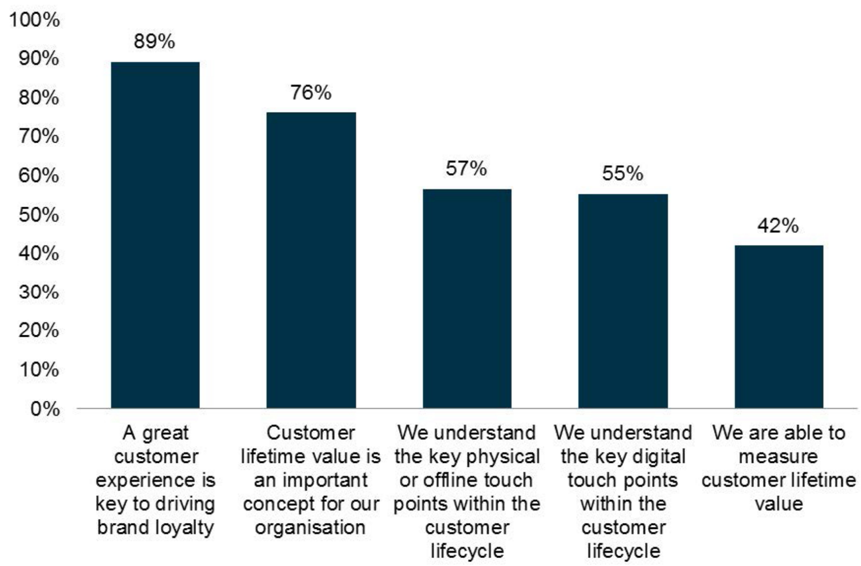

Figure 2.1: CLV-related statements, Source: (Econsultancy, 2014)

Figure 2.1 shows what proportion of respondents either agreed or strongly agreed with given research statements. The most important fact from the customer lifetime value point of view is that 76% of respondents agree that CLV is an important concept for their organization but only 42% claim their ability to measure it. Based on the research results, respondents directly link customer experience and brand loyalty (89%). As customer loyalty (client attrition) is one of the key pillars of the CLV (Gupta et al., 2006), one can assume that customer experience and CLV are also strongly dependent.

Current key areas of CLV as of 2014 and the most likely future ones are shown in figures bellow respectively.

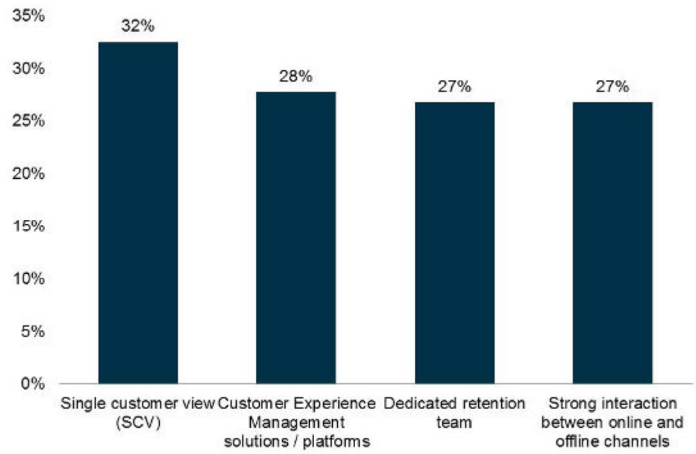

Figure 2.2: Four most effective tools for enhancing CLV today, Source: (Econsultancy, 2014)

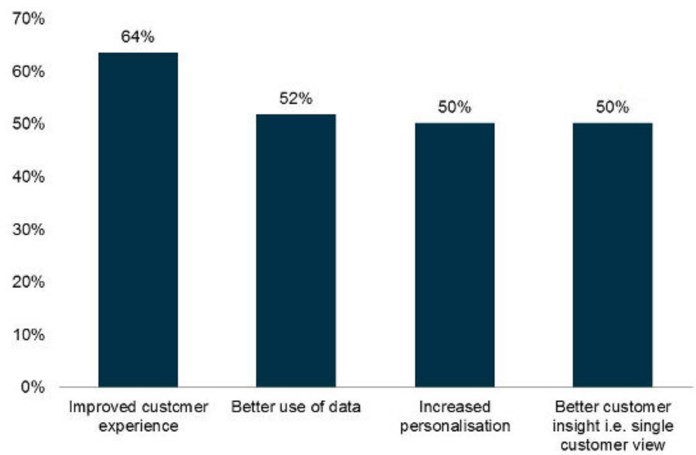

Figure 2.3: Four areas most likely to increase CLV in the future, Source: (Econsultancy, 2014)

Both charts in figures 2.2 and 2.3 point out Single Customer Value (SCV) as an effective tools for enhancing CLV. On the contrary, awaited areas in the figure 2.3 focus on better leveraging data (e.g. improved customer experience and increased personalization) whereas the current chart emphasizes the importance of employees added value (e.g. dedicated retention team and interaction between online and offline channels). Overall, there is a visible trend of digitalization among these charts.

According to (Econsultancy, 2014) the CLV opportunities in the financial services industry are firstly building trustworthy relationships, secondly adding financial certainties to customers' lives. These are essential assumptions to prevent customers' switching to competitors. To enhance CLV the researchers suggest increasing the customer interaction rate as in financial services are usual annual or even less frequent purchases.

Another study focusing on human behavior and the impact of national culture and the economic dimensions of a country on CLV was performed by (Kumar & Pansari, 2016). As the study was restricted to 30 countries and one retailer only, the scope is limited. Data used to train a choice model (probit) consisted of transactional data, cultural data and a macroeconomic variable (GDP per capita). Cultural data, gathered from Hofstede's research[2], included 5 cultural dimensions, namely Individualism, Uncertainty Avoidance, Long-Term Orientation, Masculinity and Indulgence.

Summary of supported hypothesis (Kumar & Pansari, 2016):

(i) **Individualism:** The higher the individualism in a country, the greater is the positive impact on multichannel buying on the contribution margin. The lower the individualism in a country, the greater is the positive impact of cross-buying on contribution margin. The lower the individualism in a country, the greater is the positive impact of owning a loyalty card on contribution margin. The higher the individualism in a country, the greater is impact of the inverse U-shaped relationship of returns on contribution margin.

(ii) **Uncertainty Avoidance:** The lower the uncertainty avoidance in a country, the greater is the positive impact of multichannel buying on contribution margin. The higher the uncertainty avoidance in a country, the greater is positive impact of cross-buying on contribution margin. The higher the uncertainty avoidance in a country, the greater is the positive impact of owning a loyalty card on contribution margin. The lower the uncertainty avoidance in a country, the greater is the impact of the inverse

[2]More information about the Hofstede's research and the data itself can be obtained from https://www.hofstede-insights.com/country-comparison/

U-shaped relationship of returns on contribution margin.

(iii) **Long-Term Orientation:** The higher the long-term orientation in a country, the lower is the positive effect of multichannel buying on contribution margin. The higher the long-term orientation in a country, the greater is the positive impact of cross-buying on contribution margin. The higher the long-term orientation in a country, the greater is the positive impact of owning a loyalty card on contribution margin.

(iv) **Masculinity:** The lower the masculinity in a country, the greater is the positive impact of cross-buying on contribution margin.

(v) **Indulgence:** The higher the indulgence in a country, the lower is the positive effect of multichannel buying on contribution margin. The higher the indulgence in a country, the greater is the positive impact of cross-buying on contribution margin. The lower the indulgence in a country, the greater is the positive impact of owning a loyalty card on contribution margin.

(vi) **Economic Factors:** The higher the country's GDP per capita, the higher is the contribution margin.

The study proofed the importance of both the cultural and the economic dimensions of a country for maximizing firm profits. The model results indicated that cultural dimensions affect the relationship between CLV and purchase frequency or contribution margin and that the country's economy has a direct impact on both purchase frequency and contribution margin. (Kumar & Pansari, 2016)

Based on these results, modeling CLV in a local context might be abstracted from cultural diversities as the nation can be regarded as homogeneous. However, CLV projection of worldwide scope should consider cultural differences as one of important characteristics besides the transaction data and the macroeconomic situation.

2.2 Frameworks and models used for CLV modeling

(Gupta et al., 2006) propose simplified Conceptual Framework for Modeling CLV (figure 2.4) regardless of the industry specifics. Authors directly link marketing programs with CLV, Customer Equity (CE)[3] and the firm value. The CLV approach has several advantages compared to traditional marketing metrics such as brand awareness, customer attitudes, or even sales volumes and market share. Short-term oriented marketing actions aimed at traditional metrics may have negative impact in long-run on the firm's profitability and value. Additionally, researches (Gupta et al., 2006) claim that based on other empirical studies CE[4] affects the stock price.

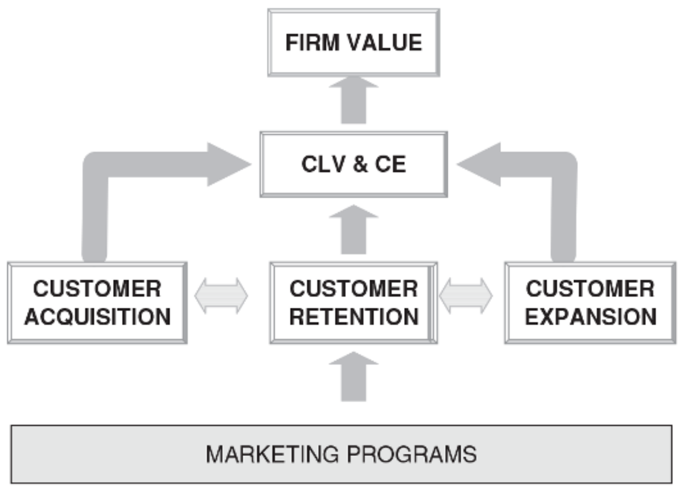

Figure 2.4: Conceptual Framework for Modeling Customer Lifetime Value, Source: (Gupta et al., 2006)

Overview of the most used techniques to predict CLV

According to (Gupta et al., 2006), six modeling approaches typically used by researches exist: RFM models, probability models, econometric models, persistence models, computer science models and diffusion/growth models. These approaches will be further described in

[3]Customer Equity is defined the chapter 3.2.

[4](Gupta et al., 2006) define CE as CLV of current and future customers.

detail.

(i) **Recency-frequency-monetary (RFM) Models**

RFM models can be classified as simple behavior models as they are based on recency of the last customer's purchase, frequency of purchases and the monetary value of past orders. This kind of models has been used mainly in direct marketing with the objective to improve low response rates (typically 2% or less) (Gupta et al., 2006). Compared to models based on the customer's demographics only, RFM models provide better prediction accuracy.

The outputs of RFM models are groups of customers "cells" based on three dimensions - Recency, Frequency and Monetary value. A simple suggested solution can be to divide each of these variables to n buckets with the comparable size (number of customers). A three-dimensional matrix of size $n \times n \times n$ is created and the most profitable customer cohorts can be identified.

The RFM model's ability to score customers based on their behavior to target marketing campaigns in the short-term has been proven by researches (Gupta et al., 2006; Venkatesan & Kumar, 2004). Whereas the usage of such models for CLV modeling is limited mainly due to following issues: RFM models predict the customer's behavior for the next period only (CLV must be predicted for a certain time horizon, e.g. 24 months), secondly the reliability of these models is affected by the lack of other demographic variables and finally customers past behavior might be a result of launched marketing campaigns. (Gupta et al., 2006)

(ii) **Probability Models**

A probability model is a representation of the world in which observed behavior is viewed as the realization of an underlying stochastic process governed by latent (unobserved) behavioral characteristics. The key assumption of probability models is that the behavior across the population can be described by some probability distribution. (Gupta et al., 2006)

To the most common probability models in terms of CLV modeling belongs the Pareto/NBD model (Verhoef & Donkers, 2001; Bas Donkers, 2007). Pareto/NBD model's assumptions are following: (Gupta et al., 2006)

- There are 2 customer states only, the customer is either "alive" (performed a transaction in the recent history) or permanently inactive.
- The number of transactions performed by an active ("alive") customer is characterized by a Poisson distribution.
- The distribution of transaction rates across customers follows a gamma distribution.
- The active customer's lifetime can be described by the exponential distribution.
- Heterogeneity in dropout rates across customers follows a gamma distribution.
- The transaction rates and the dropout rates vary independently across customers.

The Pareto/NBD model can be regarded as a base model for CLV computation in various industries. Unfortunately, in terms of long-contractual settings this model is not suitable, the assumption of transaction rates distribution is violated. However, this approach was successfully used by many authors in retail industries, e.g. (Cheng et al., 2012; Pablo Casas-Arce, 2017; Chamberlain et al., 2017).

(iii) **Econometric Models**

The econometric and probability models have many characteristics in common, for instance the churn prediction might be like Pareto/NBD models. The main difference between these models is the usage of more general hazard functions. Typically, the CLV is estimated from number of models aimed specifically at customer acquisition, retention and expansion (cross-sell, up-sell).

(Gupta et al., 2006) divide econometric models into 2 classes. First class of models tries to fit a function of hazard rate, depending on the assumption of the error term, the most used models are logit and probit models. To model the flow of customers among industry competitors or customer states, it is more appropriate to use the second class of models which allow these switches. Generally, these models are based on Markov chain or a more complex manner - Markov Decision Process. Such models in a combination with the hazard rate models used as predictors are especially valuable in the long-lasting contracts e.g. banking, insurance and telecommunication industries (Peter Paauwe, 2007; Michael Haenlein, 2007; Mzoughia & Limam, 2014).

The usage of econometric models from the company's perspective is very broad as they generally perform well in customer relationship management. For example, Michael Lewis in (2005) performed a dynamic pricing study where he found out that new customers are more sensitive to price changes compared to current ones. According to that fact he suggested a series of reducing discounts instead of switching from the discounted service to the regular price, i.e. 75% discount on the subscription within the first period, 50% in the second, 25% in the third and regular price further on.

(iv) **Persistence Models**

Persistence models are focused on modeling customer's behavior based on time-series data. In majority of cases the Vector Autoregressive (VAR) models are used. The advantage of these models is their ability to study what is the impact of a change in one variable to other variables or the overall system's behavior, i.e. simulate the demand shocks related to marketing campaigns and the influence on other variables in the same or even delayed period.

Generally, persistence modeling (VAR) consists of three separate steps: (Gupta et al., 2006)

1. Examine the evolution of each system's variable over time.
2. Estimate the VAR model (typically with the least-square method). The VAR(p) formula for 3 time series model (number of customers acquired by marketing actions MKT, number of customers acquired by Word of Mouth WOM and the firm's

performance value $VALUE$) according to (Vraná & Jašek, 2015) is following:

$$\begin{pmatrix} MKT_t \\ WOM_t \\ VALUE_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} + \sum_{l=1}^{p} \begin{pmatrix} a_{11,l} & a_{12,l} & a_{13,l} \\ a_{21,l} & a_{22,l} & a_{23,l} \\ a_{31,l} & a_{32,l} & a_{33,l} \end{pmatrix} \begin{pmatrix} MKT_{t-l} \\ WOM_{t-l} \\ VALUE_{t-l} \end{pmatrix} + \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix} \quad (2.1)$$

where t stands for time, vector (c_1, c_2, c_3) contains the constant terms and vector $(e_{1,t}, e_{2,t}, e_{3,t})$ involves the error terms. (Vraná & Jašek, 2015) have shown that relationship between variables can be described by the VAR model. Namely:

- *direct effects* of acquisition on the firm's performance
- *cross-effects* between two types of customer acquisition
- *feedback effects*, which show how the firm's performance in period t is affected by the one in $t-1$
- *reinforcement effects* when the time series values affect each other, e.g. customers gained by Word of Mouth spread the feedback and therefore influence the future acquisitions

3. Derive the impulse response functions with respect to the VAR model estimates.

The VAR model can perform well as authors (Vraná & Jašek, 2015) proofed by their research in the online retailer. How is presented in the figure 2.5. The VAR models generally (if correctly calibrated) have a great ability to predict the future values as the underlying variables are in the form of time series.

However, online retail is specific in the way how precisely one can measure the impact of marketing campaigns as the prior websites and actions are known due to saved cookies and other tracking systems e.g. Google Analytics. Implementation in retail banking is a far more challenging task as there is no straight-forward method how to estimate the impact of marketing actions, furthermore the relationship within the industry is not based on purchases but more on the product usage and eventually the product acquisition.

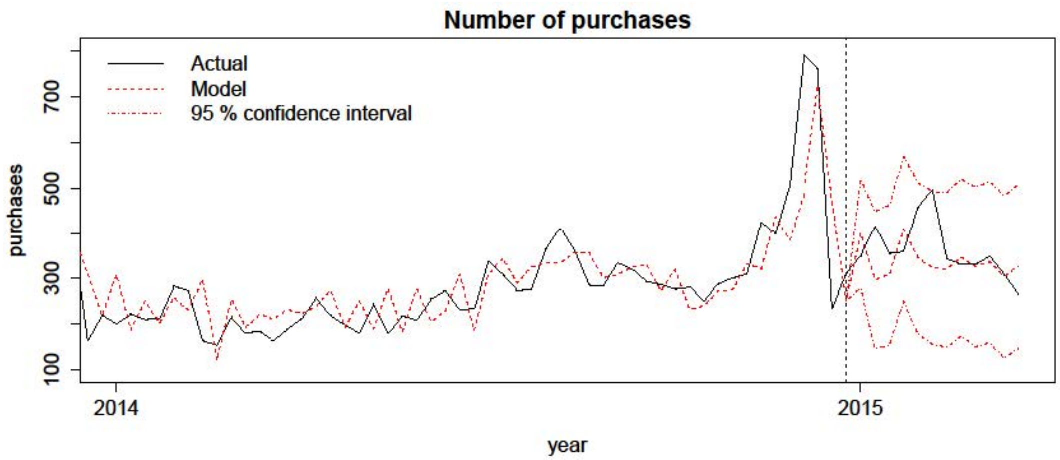

Figure 2.5: The number of purchases in an online retailer interpolated and extrapolated by the VAR model, Source: (Vraná & Jašek, 2015)

(v) **Computer Science Models**

Generally, computer science models are not of a borad usage in marketing as these models are more difficult to interpret than structured parametric models (e.g. logit, probit). However, computer science models such as neural networks, decision tree models, classification and regression tress (CART) and support vector machines (SVM) mostly have higher predictive ability. Not only the accuracy of these models is exceeding, but several studies have proven that also the top-decile lift is significantly higher (Gupta et al., 2006). Bearing in mind the limited marketing budgets especially the top-deciles may generate profit of hundreds thousands dollars.

Another advantage of computer science models is the opportunity to build several models with various algorithms and later create an ensemble model. The ensemble model can use computer science algorithms again or just simply calculate a weighted average of first-level model predictions. Ensemble models are particularly useful for enhancing the model stability, secondly the prediction accuracy.

Computer science models benefit from a very large number of variables, sometimes referred as the "curse of dimensionality". Such feature space inflates the variance of the estimates, making traditional parametric and nonparametric models less useful (Gupta et al., 2006). In terms of CLV this approach is applicable in businesses with a long contractual history e.g. banking, telecommunication. On the contrary, while having only a few variables (e.g. online retailers recency-frequency-monetary data), statistical or persistence models are more suitable.

(vi) **Diffusion/Growth Models**

Compared to previous models, diffusion/growth models do not predict the probability of acquiring a particular customer, rather they use aggregate data to predict the number of customers that the company may acquire. This principle is based on the Roger's Diffusion of Innovation theory as showed in the figure 2.6 which assumes a normal distribution of innovations among the population, thus the cumulative function creates an S-curve. (Rogers, 2003) claims that in the population there are only 2.5% of Innovators, 13.5% of Early Adopters, 34% of Early Majority, 34% of Late Majority and finally 16% of Laggards.

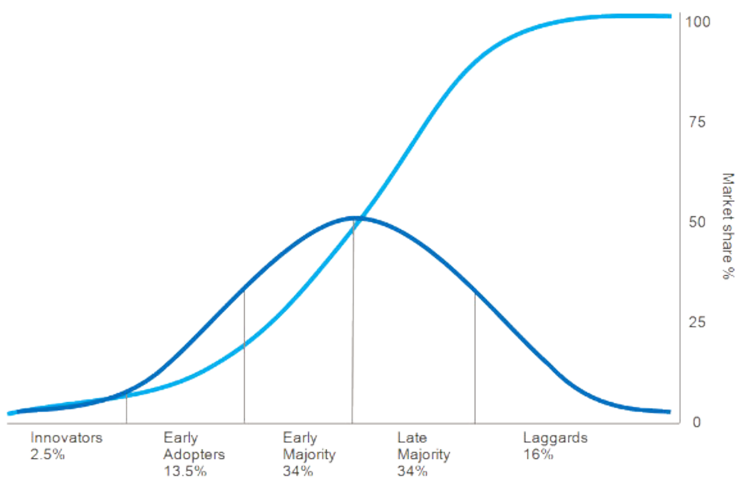

Figure 2.6: Diffusion of Innovation Model, Source: (Rogers, 2003)

Authors (Cheng et al., 2012) applied the growth models to predict the number of online Australian shoppers over time. The comparison of the models predicting first-time shoppers is visualized in the figure 2.7 and cumulative function in 2.8. (Cheng et al., 2012) have proven that in the case of new services or product launches on the market, growth models perform better than logistic models.

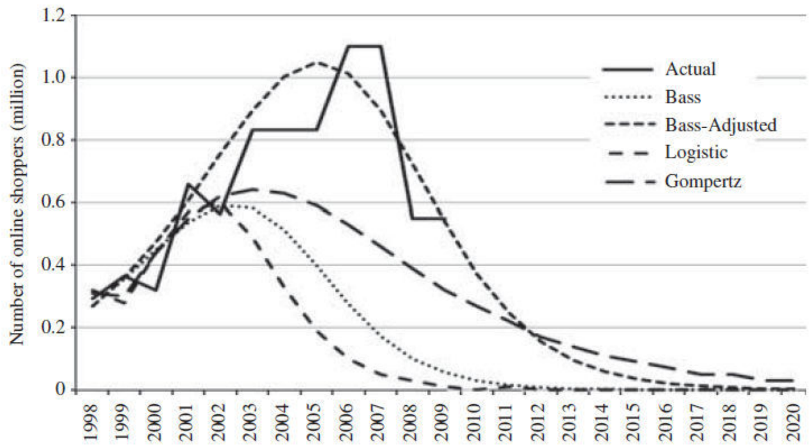

Figure 2.7: Fit of the growth models predicting number of Australian online shoppers (1998-2002), Source: (Cheng et al., 2012)

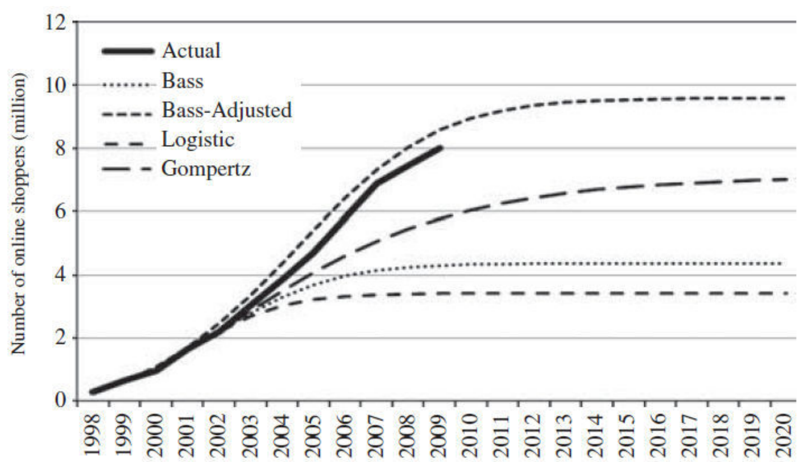

Figure 2.8: Fit of the growth models predicting cumulative number of Australian online shoppers (1998-2002), Source: (Cheng et al., 2012)

Interestingly, according to (Gupta et al., 2006) diffusion models may be used also for loses prediction while having an assumption that leaving customers spread their attitudes among others (negative word of mouths) and due to this fact current customers become less loyal.

The diffusion/growth models fit the aggregated numbers very well therefore they are used for evaluations of products, services or even whole companies (Gupta et al., 2006). On the other hand, deriving individual values (e.g. CLV) for a particular customer is hardly possible and if so only with great amount of approximation.

A comprehensive framework combining econometric models (logistic regression) and computer science models (regression tree, random forest and neural network) was proposed by (Chamberlain et al., 2017). The framework is displayed in the figure 2.9. Authors examined factors influencing CLV in the online fashion industry. Their work is especially valuable for the integration of deep learning algorithms (neural networks) with the ordinary machine learning pipeline. Not surprisingly the most important variables for the CLV modeling were Purchase history (60% overall importance) and Web/app session logs (34.5% overall importance). Variables with only minor importance were Customer demographics (7.8%) and Returns history (1.7%).

The low importance of Customer demographics may be explained by lack of precise predictors as online retailers usually do not record information about categories such as age or education. However, some features might be obtained from the customer's location based on device's GPS or IP address and delivery address. Location data in statistical context may reveal the estimated demographic information (e.g. social class, ethnicity, religion, work-commuting time etc.). The demographic profiles may allow retailers to better target the customers with relevant offers and hence increase customer lifetime value. Despite the expected information gain hidden in geographic data, authors (Chamberlain et al., 2017) moved their focus towards Web/app session logs.

The uniqueness of web/app behavior data lies in its variety and thus difficulty of extracting handcrafted features for modeling. To mitigate this issue authors (Chamberlain et al., 2017) decided to use neural network models powered by Google Tensorflow[5]. These fea-

[5]More information about the technology Google Tensorflow can be found at:

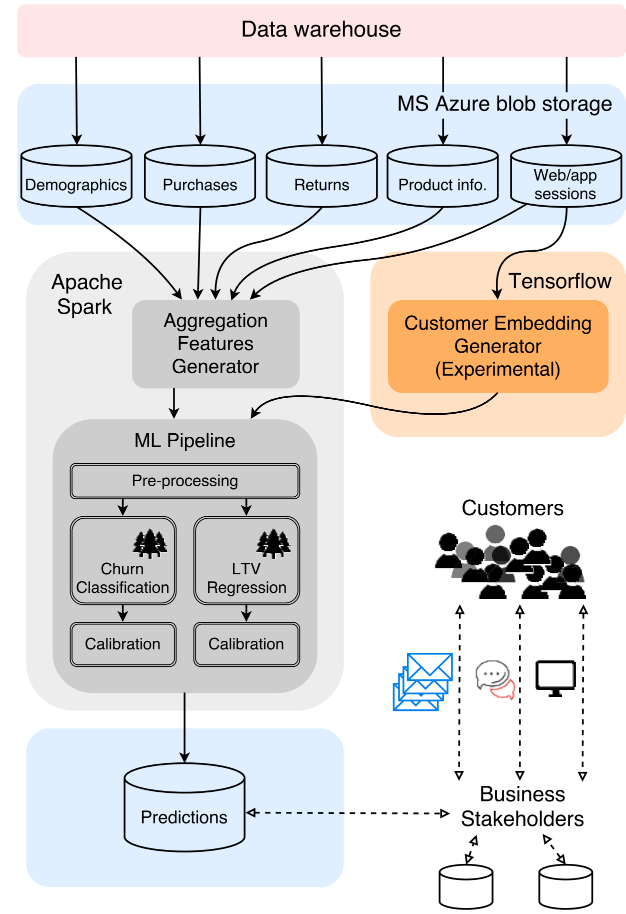

Figure 2.9: CLV Framework with Customer Embedding Generator, Source: (Chamberlain et al., 2017)

tures were pooled together with other traditionally handcrafted ones and further processed for the Churn classification and CLV regression.

Another part of the proposed framework worth pointing out is the calibration. To achieve consistent results, authors (Chamberlain et al., 2017) calibrated outputs of both Churn classification and CLV Regression. They used a simple linear regression technique to perform the calibration. According to (Chamberlain et al., 2017), this approach has 2 main advantages: (1) the model becomes more robust to the existence of outliers and (2) average values of obtained predictions over a set of customers match the observed ones more

https://www.tensorflow.org/

accurately.

A framework based on the Markov Chain model from the group of Probability models was applied by (Cheng et al., 2012). The framework was designed to fit a car-repair industry, but again it can be used in other industries with repetitive purchases or long-lasting contracts. The framework has 3 main pillars as can be seen in the figure 2.10, namely Lifetime prediction, Profit prediction and Behavior prediction. The Markov chain model is then built based on outputs of these components and calculates the estimated lifetime value.

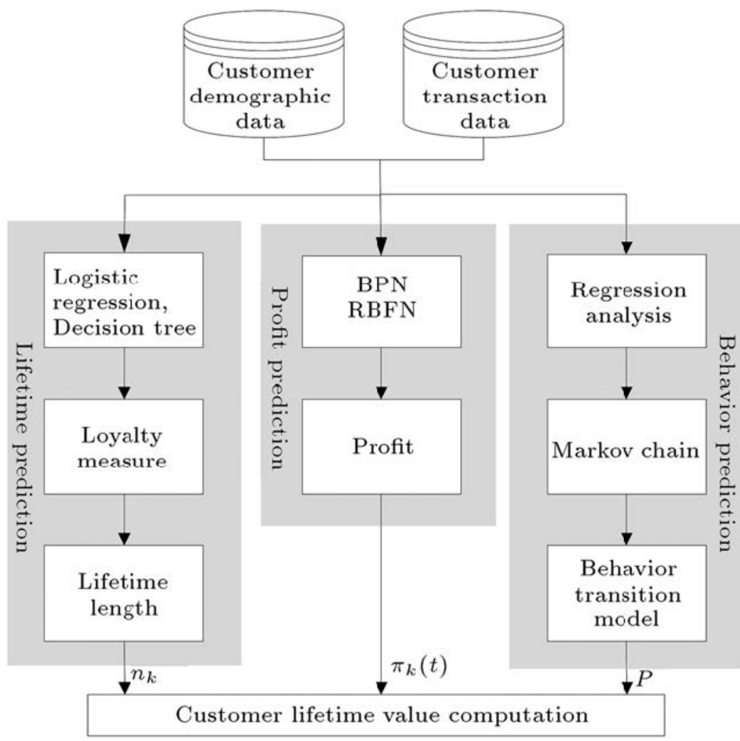

Figure 2.10: CLV Framework based on Lifetime, Profit and Behavior prediction, Source: (Cheng et al., 2012)

The Cheng's study is valuable more for the framework structure than the results itself because authors did not examine relationship specifics of the industry such as mileage and service inspections. This was used to model ordinary service visits (e.g. oil exchanges, brake replacements) as well as unplanned ones (e.g. damages caused by accidents, sudden car part breakdowns), the data structure and quality is an additional constraint. The final Markov chain model involved only one variable (number of visits by the customer over year), which is an unsatisfactory result.

2.3 Modeling CLV and its usage in various industries

Many studies show that customers have different attitudes towards marketing channels which affect the propensity of acquisition of the marketed product, and consequently influence CLV nonlinearly. Authors (Venkatesan & Kumar, 2004) claim that customers who are selected on the basis of their CLV provide higher profits in future than do customers selected on the basis of several other customer-based metrics.

Rozek and Karlicek (2014) pointed out the difference between the CLV oriented approach and the profit-oriented approach on the Product Quality Game (Game Theory model). This non-cooperative game has 2 players (the customer and the company). The game scenario is as follows: the company manufactures and offers a product for a fixed price, it can choose whether to produce high-quality products (which lowers the company's profit) or low-quality products (which increases the company's profit). The customer has 2 options as well – either buy a product or not. The customer is limited by the fact that he or she does not know the product quality in advance (before the actual purchase). The possible payoffs of the customer's and company's strategies are mentioned in the table 2.1.

Table 2.1: Product Quality Game - Payoff matrix, Source: (Rožek & Karlíček, 2014)

	Customer's strategies and payoffs		
Company's strategies and payoffs		**To purchase**	**Not to purchase**
	HQ product	5 / 5	0 / 0
	LQ product	10 / -5	0 / 0

The Nash equilibrium[6] under such conditions lays in the combination of low quality

[6]The Nash equilibrium point is an n-tuple such that each player's mixed strategy maximizes his payoff if the strategies of the others are held fixed. Thus, each player's strategy is optimal against those of the others. (Nash, 1951) In other words the Nash equilibrium is such a combination of strategies that neither of the players can increase their payoff by choosing a different strategy.

production from the company's side and decision not to purchase from the customer's side. However, neither the customer nor the company will profit from this strategy, which leads into change in the company's behavior. The company has generally two options: (1) pretend that it produces high-quality products while manufacturing the low-quality ones (raising its marketing expenditures) or (2) manufacture high-quality products and thus diminish its profit. (Rožek & Karlíček, 2014)

The first option puts the company into the situation of moral hazard, on the other hand while having an assumption of non-repetitive purchases from the particular customer and low information about the perceived quality spread among customers, this behavior may be very advantageous. For example, this scenario is typical for various businesses in the touristic destinations. Nonetheless the majority of companies cannot trick customers in such way in long time horizon and therefore the only sustainable strategy from the company's side is to offer high-quality products. The customers then have no need to terminate the relationship and go to the competitor.

Implications from the Product Quality Game for the CLV modeling are undoubtable – once acquired customers will stay only in the fair *"win-win"* relationship, where they will be treated with respect. On the contrary according to (Rožek & Karlíček, 2014), 20% of the customers create from 150% to 300% of the company's profit, from 60% to 70% customers are on the break-even (their CLV equals to zero) and from 10% to 20% of the customers lose from 50% to 200% of the company's total profit[7]. Authors therefore suggest spending marketing expenditures reasonably and focus on the high profitable customer groups.

Bas Donkers (2007) studied the capabilities of a range of models to predict CLV in the insurance industry, e.g. a status quo model, a Tobit II model, choice models, and duration models. Even though the targeted time horizon for the CLV prediction was 4 years, the more complex models did not far outperform the simplest ones (the status quo model which predicts the next period customer's profit to be the same as the current one). This might

[7]The relationship between customer percentiles and the cumulative profit can be seen in the figure 3.1.

be caused by insufficient feature space since there were only data about insurance type, loyalty program, purchase history, cancellation history, and contract duration available. The status quo model performed well particularly because of relatively large number of customers that do not change their purchase behavior (insurance types) over time. From the business perspective the (Bas Donkers, 2007) work revealed which factors affect the customer retention rate (the probit model) and the cross effects at the insurance level. The highly correlated insurance types may lead to cross-sell initiatives as one can assume similar customer needs.

Another study by (Peter Paauwe, 2007) was performed with the aim to estimate the CLV of customers in an e-commerce environment. Although several other researches mentioned in the thesis focus on the same industry, this study is specific considering the market segment restriction. Author's oriented on the sells of ink cartridges for ink-jet printers, therefore the customer's behavior is more likely a long-lasting relationship as the mean period between purchases is 2.7 months. Due to this fact (Peter Paauwe, 2007) decided to use the Decision Tree Markov Chain (DTMC) model. The proposed model consists of 2 phases, (1) Decision tree step - the estimation of segments (customer states) using a CART tree and (2) Markov chain step - the computation of a transition matrix based on the estimated customer state changes in the training period, a prediction of the client base structure after n periods and discounting the observed values. Author's admit that different models outperform the DTMC model (e.g. RFM models) in CLV predictions, nevertheless they highlight DTMC model's ability to measure marketing effort effects and its great interpretability to business users due to clearly defined customer states. Though authors mention that the most limiting factor was the lack of demographic data. Therefore number of discovered segments was probably not sufficient and lead to distortion of Markov chain estimators.

Interesting application of lifetime value mentioned (Sutton & Barto, 2016) in the case study aimed at personalized web services. Their assumption was to use Markov Decision Process to improve the *click-through rate* (CTR), which he defines as the ratio of the total number of clicks all users make on a webpage to the total number of visits on the page. The underlying idea is that customers who have already seen the advertisement on the webpage are affected by that fact (Markov states). Customers demonstrate their attitude by clicking the add, hoovering over it or reading the page without any action. A webserver responds to the customer's behavior with an appropriate action based on the customer's state. The lifetime value metric used in the web analytics is not directly transformable to the Customer Lifetime Value, but might be a valuable concept in designing the company's webpages and revealing customer's reactions, e.g. new product launches. The idea of a sequence of offers following a predefined policy is also favored by one of the market leaders Adobe that implemented this feature in the Marketing Cloud.

Summary

The estimation of CLV value is a very current topic mentioned in a broad range of researches focused on marketing, computer science or even sociology. Especially in highly competitive retail banking – industry specific for the volumes of customer data – CLV prediction and optimization emerged to be a critical concept to retain the performance and market share. Only few of the discussed models provide the results on customer level, which is a crucial property for further CLV optimization. However, use cases when an aggregated customer portfolio lifetime value is sufficient exist (e.g. business valuation), therefore these models were mentioned to provide a thorough overview of CLV approaches.

Chapter 3

Campaign Optimization in Constraints of Retail Banking

The aim of this chapter is to describe the retail banking environment and its marketing initiatives, metrics used to evaluate the marketing performance and finally introduce the market leading solutions from vendors such as Adobe, SAS, or Oracle. A comprehensive framework is proposed and discussed at the very end of the chapter.

3.1 Banking industry specifics

The financial services are getting more and more complex as technical capabilities allow targeting not only the "mainstream" clients with average needs but extending the product portfolio to offer a tailored product combination for each individual client. Consequently, this behavior increased the requirements on knowledge of financial advisors and therefore the time necessary to prepare new employees for the desired job.

Whereas in corporate banking each client's needs are so specific that automation of these processes would be too costly or even impossible, in retail and potentially Small and Medium-sized enterprise (SME) banking clients can be clustered into groups (segments) which are internally homogenous and externally heterogenous, i.e. clients in each segment have similar characteristics to each other but different from those in other segments. These segments can be individually treated by the most suitable strategies which fit client's needs. The predefined strategies help reduce employee's training time and allow financial advisors to offer the optimal products and services. The banking domain expertise is not an objective of this thesis and therefore the corporate banking is not further considered.

The banking environment can be distinguished from other industries by many different characteristics based on the evaluator's point of view. From the marketing perspective three main factors can be identified, namely (1) the regulation restrictions imposed on the industry, (2) the indispensable trustworthiness of financial institutions and finally (3) the relationships between the businesses and their clients are long-lasting, sometimes even life-long lasting and may be of a significant influence on the client's lives.

- **Regulations:** The data exchange between the bank and third-party businesses is particularly limited by the legal restrictions as the data security is an important concern. Banks have to comply with not only banking specific regulations e.g. Sarbanes-Oxley Act (SOX), but also the e-privacy protecting laws, for example General Data Protection Regulation (GDPR). For instance, when the bank needs to enrich its internal data sources in e.g. customer's behavior or risk score, it must agree with its partner on the cryptographic algorithm that will be used for data anonymization, which ensures that any intermediate party will not be able to reconstruct any personal information. Usually one-way cryptographic hash functions (e.g. SHA1, SHA256) are used for such tasks. Moreover, the bank is not allowed to share internal client's data to business partners, which hardly affects data monetizing techniques well developed in other sectors of economy (e.g. offering of complimentary services through the business partners).

- **Trustworthiness:** The market share is relatively stable as the economic barrier to entry is very high. New comers must demonstrate the strong financial background (initial capital), fulfill the central bank's conditions to obtain a banking license and most importantly work on the reputation to gain their credibility as they operate with people's savings. Building trustworthy relationships between banks and their clients is a long-lasting process, on the other hand it can be heavily affected by fraudulent or suspicious activities. The damaged reputation in such cases may lead up to a bankruptcy (e.g. Lehman Brothers crisis in 2008).

- **Duration of relationships:** Compared to other industries, bank clients cannot choose from such a broad variety of competitors, which is definitely one of the determining factors. However, if the bank successfully meets client's expectations and needs – from day to day transactions to debt financing and investment management – the probability of client's churn decreases to a minimum level. Therefore, a client staying throughout the whole life in a relationship with one particular banking institution is not an exceptional behavior. Interestingly as (Gupta et al., 2006) claims in the study, this cannot be generalized at the whole financial industry. Even though insurance companies operate with client's capital as well, the fluctuation rate among competitors is significant and leads to retention activities.

3.2 Customer Metrics

Several kinds of metrics may be used for evaluation of customer's performance beginning from simple one-time profit oriented, over client's needs penetration up to more complex ones such as Customer Lifetime Value (CLV). As mentioned in the study (Abdolvand & Albadvi, 2012), a critical requirement in performance management is the closeness of the metric to the actual client performance. In a short-run the profit-oriented metric can be sufficient for the performance evaluation. However, as already discussed in this chapter, banking industry is firstly specific in the duration of relationships and secondly the behavior leading to profit generation is sometimes not utterly ethical which is related to the level of client's financial literacy. Regulators try to protect end clients by penalizing financial institutions for an inappropriate advisory (e.g. high risky and profitable investment opportunities sold to clients not fully understanding the product and potential loss). On the other hand, the situation is not straight forward as the aim of majority of enterprises not only in banking industry is to generate profit to their owners or shareholders.

The customer metrics evaluated from economic and managerial perspective (abstracting from ethical aspects) with its pros and cons are summarized below.

- **Financial Gain - Profit, Margin** The main advantage of this metric is its simplicity and direct financial impact. From the managers point of view clear objectives can be set (e.g. achieve total profit of $ 10 Million quarterly). The motivational component in the employee's salary can be calculated as a certain percentage from the total profit or turnover. On the contrary this strategy may lead to suboptimal solutions offers, for instance financial advisors may decide to promote a product with higher financial commission or customize the product parameters such as the total amount or monthly installment in a way which is not relevant to client's needs. Once the client finds out that there are more fitting products on the market or a new competitor occurs with an aggressive strategy, the probability of a client's churn rapidly increases in such cases. The loss cumulated based on these churned clients is then likely to surpass the short-term profits.

- **Contract - Turnover, Count, Usage** Whereas the previous strategy is mainly applicable to one-time product acquisitions (e.g. cash loan, investment), it is not useful for contract types such as checking account or credit card account. The client's activity is then measured as an account turnover, a count of card transactions per month, or the remaining account balance etc. In general, these metrics are related to financial gain as one can assume that from every card transaction the bank gets a small commission, similarly the higher is the sum of incoming payments to the account, more services are used, and the final amount of fees is higher. To motivate clients to use the account as their main checking account, the banks usually offer extra deals – such as premium interest rates on both checking and saving accounts if the card payment count is higher than a certain number, giving extra reward points to benefit programs for the incoming transactions, or the frequent flyer deals for credit card users etc. Unfortunately, these initiatives are very frequently misused as client adapt quickly and typically do not achieve the set target.

- **Asset Value - Customer Lifetime Value (CLV), Customer Equity (CE)** Whereas the previous metrics can be described as mainly product oriented, CLV is a completely different approach. As mentioned in the study conducted by (Rožek & Karlíček, 2014), the CLV approach shifts the marketing focus from the one-way marketing communication in favor of a mass production towards customers, resulting in omnichannel bi-directional communication and personalized offering. This change towards customer-centric viewpoint is sometimes referred as a redirection from transactional to transformational marketing (Rožek & Karlíček, 2014). By the transformation in such context authors mean the long-term relationship orientation, which is especially applicable in the context of retail banking. Even though as mentioned in (Rožek & Karlíček, 2014) the CLV approach is nothing particularly new and the CLV calculation is relatively simple, in the market research conducted by (Econsultancy, 2014): 42% of companies were not able to measure it.

  The CLV metric has its roots in investment management. Customers are treated as firm's assets; each customer relationship requires an initial investment (acquisition costs) which should produce respective returns. The cash flows are spread over multiple periods and therefore discounted based on the interest rate. (Rožek & Karlíček, 2014)

> **Customer Lifetime Value (CLV)** can be defined as the sum of all future discounted profits coming from the customer relationship, i.e. all revenues minus all costs related to the customer interaction with the firm (Rožek & Karlíček, 2014).

The calculation formula for each customer's CLV is as follows:

$$CLV = \sum_{t=1}^{n} \frac{p_t \times r_t}{(1+i)^t} - AC = \sum_{t=1}^{n} \frac{(R_t - C_t) \times r_t}{(1+i)^t} - AC \tag{3.1}$$

Where

- p_t = profit in period t
- r_t = probability of customer purchase in period t
- i = discount rate
- AC = acquisition costs
- R_t = revenue in period t
- C_t = costs in period t

As described in the equation 3.1, the CLV calculation has two major components: *Profit* and *Probability of customer purchase (in service oriented industries - probability of not terminating the contract).* Both variables represent a time series, the modeling approach is described in chapter 5.

In order to get the current value of future cash flows it is necessary to divide the estimated income after n periods by the discount rate. The discount rate may be estimated based on company's internal standards for asset evaluation, inflation rate, credit interest rates etc. Usually the discount rate is set annually, therefore it must be decompounded reflecting the length of one period. Assuming the one month period and 2% annual discount rate, effective interest rate is $i = \sqrt[n]{(1+I)} - 1 = \sqrt[12]{(1+0.02)} - 1 \approx 0.1652\%$ where n is the number of periods per year and I is the annual discount rate.

To provide generally acceptable CLV calculation formula, the acquisition costs were taken into consideration. Authors (Rožek & Karlíček, 2014) define acquisition costs as costs that are invested in the customer in order to influence him/her for the initial buying behavior at the start of the relationship. This variable is primarily useful for marketing strategies with directly measurable impact, on the contrary it is hard to evaluate campaigns affecting for instance brand awareness, brand recognition (e.g. television advertisements). Those initiatives can be measured only in the aggregated manner and hardly distinguished on the customer's level. Moreover, as the thesis is focused on the existing customers, the acquisition costs will not be further considered.

> **Customer Equity (CE)** represents the total value of all of the company's customers. It is described as the sum of CLVs of all of the company's current and future customers. (Rožek & Karlíček, 2014)

$$CE = \sum CLV \tag{3.2}$$

The impact of CE on the company value has examined the team of researchers (Gupta, Lehmann, & Stuart, 2004). They found out that customers are important intangible assets and their value should be measured and managed as in any other asset. Authors (Gupta et al., 2004) also claimed that the estimates of CE were reasonably close to the market valuation, moreover the traditional investment metrics (e.g. price/earnings ratio) did not work well for the valuation of many of the firms because in the study most of these companies had negative earnings. They also revealed the contrast between the acquisition and retention elasticities (0.02-0.3% compared to 3-7% respectively), i.e. 1% improvement in retention increases the customer value by 3% to 7% whereas 1% improvement in acquisition to be only 0.02% to 0.3% change in the customer value, which also emphasizes the importance of retention activities.

The Impact of Forward-Looking Metrics

The study aimed at the motivation of employees conducted by (Pablo Casas-Arce, 2017) confirmed the CLV importance among customer management metrics. Researches developed the methodology how to evaluate CLV and implemented it into an internal Customer Relationship Management (CRM) system, while not changing the employee's motivation metrics (commissions mainly linked to short-term profitability). The CLV metric's availability resulted in a significant shift in attention toward more profitable client segments and some improvement in cross-selling. Moreover, the hypothesis of negative impact of CLV on pricing or default risk could be successfully rejected. The most significant benefits of the CLV identified by (Pablo Casas-Arce, 2017) were: (1) it may be used as an instrument to align the long-term value creation strategy of an organization with the short-term profit objectives and (2) it substitutes the lack of domain experience of branch managers with shorter tenure, enabling them to target more suitable and profitable customers. The measurable impact of the CLV implementation in this particular case was the increase of 5 percent in the value of mortgage customer.

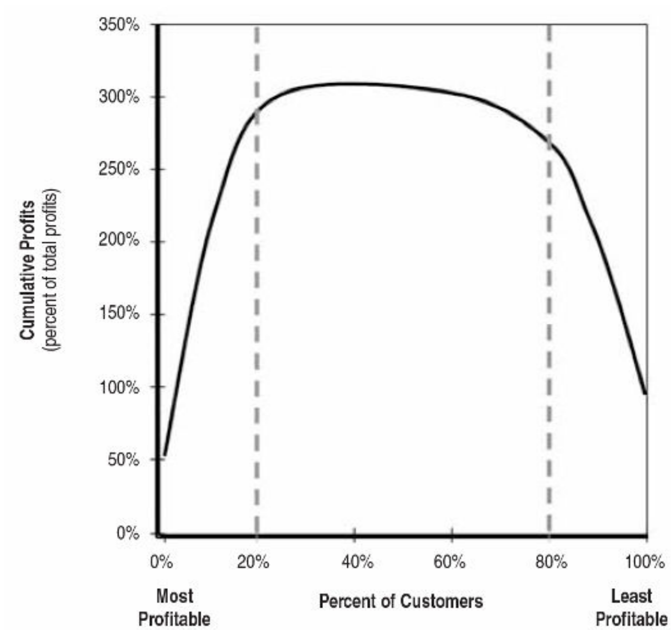

Figure 3.1: Cumulative Customer Profitability, Source: (Keiningham et al., 2006)

The previously mentioned study is utterly in accordance with the findings of (Keiningham et al., 2006). The authors identified that 20% of the most profitable customers generate up to 280% of the profit, other 60% of customers make the profit change from -20% to 30% and finally 20% of the least profitable customers cause the profit loss of 160%. The cumulative customer profitability curve is displayed in the figure 3.1.

3.3 Campaign Management Systems

Today's marketing strategy requires a comprehensive 360° customer approach, which integrates all initiatives: acquisition, cross-sell and up-sell, and retention. The simultaneous resource allocation is required to identify and meet customers' needs. The most important points to cover while considering the optimal marketing approach identified (Triplett, 2012) as:

- Optimize the investments into marketing initiatives not to waste resources and increase the customer's lifetime value.
- Determine the impact one strategy has on the other strategies.
- Consider dynamic customer lifecycles and organize the processes around them.
- Create synergies between marketing actions, find natural complements.
- Establish consistency and continuity across marketing campaigns.
- Utilize customer feedback to reinforce relationships and identify needs.

One more point should be added to the previous list regarding the compliance with national and international law. The personal data security is for instance enforced by the European General Data Protection Regulation (GDPR)[1], many restrictions are also imposed specifically on the banking institutions by the regulators.

Taking into account the above-mentioned specifics, the overall complexity of the marketing initiatives and related campaign management systems stands out. Many international vendors offer specialized solutions integrating machine learning, personalization and big data capabilities. Before mentioning some of the vendors and their tools, it is necessary to describe the data sources and define campaign itself.

[1]More information about the EU regulation 2016/679 publicly known as General Data Protection Regulation (GDPR) can be found at: http://europa.eu/dataprotection

Simplified schema of data sources and the data unification process is visualized in the figure 3.2. As was already mentioned in the section regarding banking industry specifics, the banking environment is unique in the length of customer relationship. The length of the time-series and detailed product usage create together a solid foundation for the data driven customer engagement and marketing content.

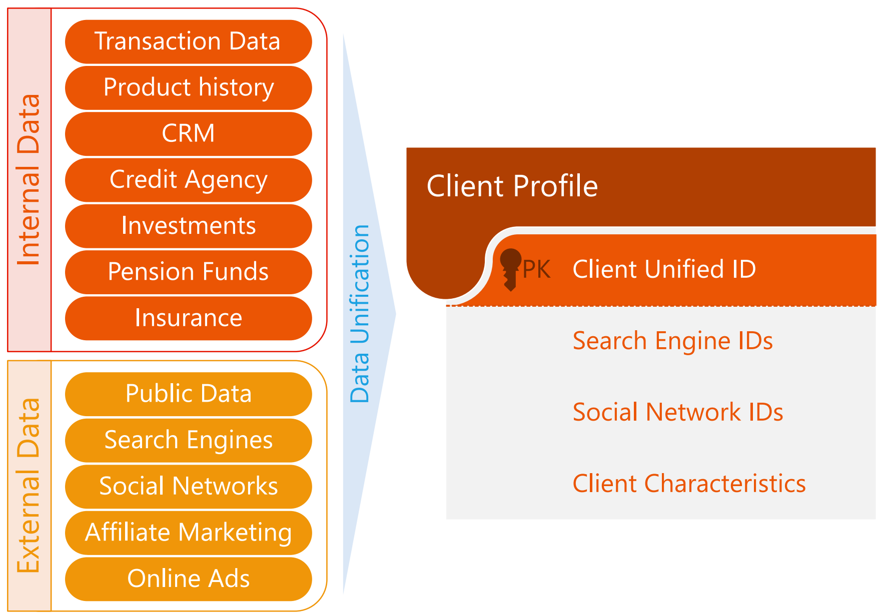

Figure 3.2: Campaign's data sources, Source: Author

A schema in the figure 3.2 illustrates how diverse the input data is. An indispensable part of the data ingestion process into the consolidated data storage is the data unification. The aim of this task is to interconnect all available client's data with a single unified identifier. In terms of marketing campaigns, usually the CRM client's identifier is the most suitable key for that purpose, on the other hand an artificially generated identifier might be chosen as well.

From the technology perspective, the client profile entity might be stored in various forms based on e.g. data type, format, and update frequency. The most common forms are for example relational, columnar or NoSQL, the storage could be centralized, distributed or even in-memory. The desired solution may consist of several platforms combining their advantages, while communicating by messaging services (e.g. Apache Kafka) and integrated by an enterprise service bus. The technology solution of campaign engines is not the aim of this thesis, therefore the least complex version of a client profile implementation is assumed.

The client profile is represented by a table in the Relational Database Management System (RDBM) with the primary key on the Client Unified ID column. The information about mapping of the Client Unified ID may be recorded in a separate dictionary, or more preferably in the Client Profile, e.g. Google Analytics ID or Facebook Audience ID is meant by such mapping. The major part of client profile data are the client characteristics. Some of those variables change on daily basis, some may represent snapshots at specific dates (typically account balances at the end of the last month), some may be even less frequent (client's risk score). Generally, the client profile data are ingested on daily, weekly or monthly basis. To ensure that the most recent data is available (necessary for example in the online interaction), real-time operational data store or similar additional layer may be used. The main requirement for this kind of data storage systems is not the ability to store large volumes of data, but the possibility of processing continuous data streams.

Academic authors and the vendors of campaign management systems define campaigns in different ways. For instance, some of the sources characterize campaign as a set of marketing offers, the others by the campaign mean a combination of marketed product and used channel. To eliminate this heterogeneity campaign is defined in the most granular way as follows.

> **Campaign** represents the unique combination of client situation, marketed product, selected client segment and the set of marketing channels. The definition is visualized in the figure 3.3.
>
> *This definition can be used only in the direct marketing environment as one of the components is the client situation. For the purpose of mass medias, the target group may be selected instead.*

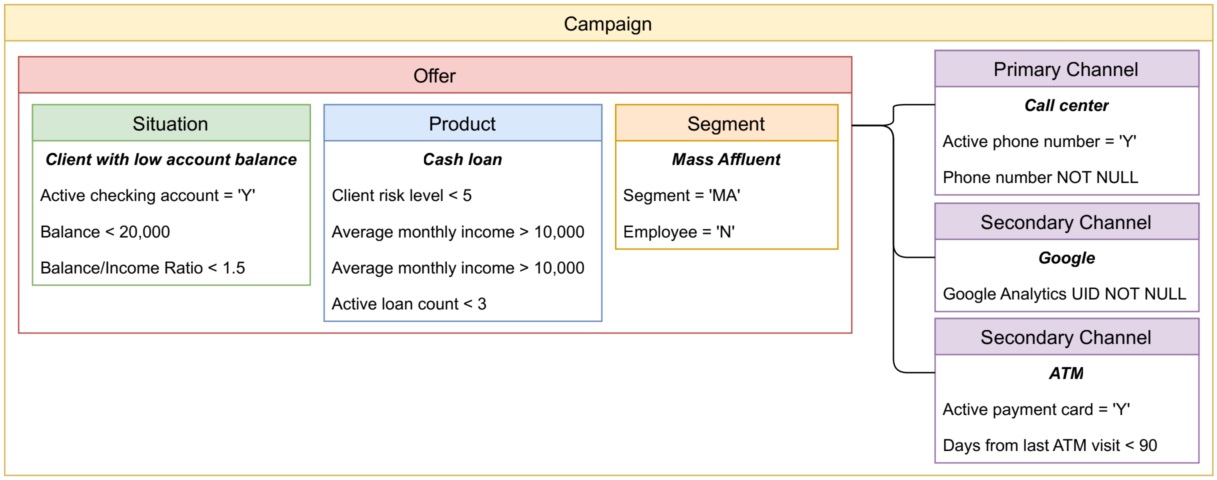

Figure 3.3: Campaign schema, Source: Author

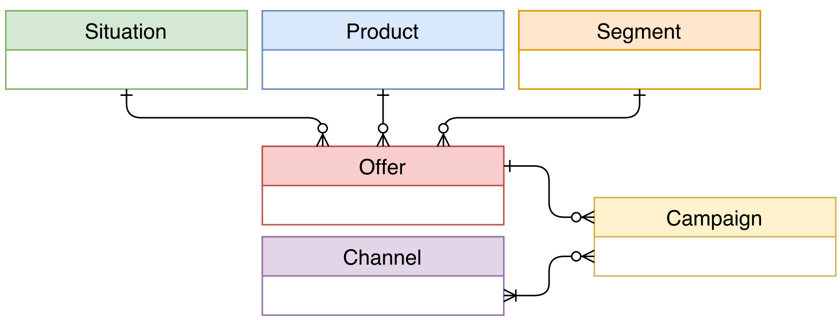

Figure 3.4: Campaign Entity Relationship Diagram, Source: Author

Figures 3.3 & 3.4 show campaign components and their relationships, each of these entities is further described. All of the attributes used in the figure 3.3 are selected from the Client Profile.

- **Offer** is a parent entity of Situation, Product and Segment. Firstly, the entity naturally represents a business need to contact a client promoting specific product while overlooking the channel (a mean of contact). Secondly, it is crucial for the campaign management system as multiple campaigns with the same Offer must be restricted only to the most optimal one.

  - **Situation** describes the client specific behavior. For some of the products it may be very general (e.g. pension fund deposit accounts suitable for anyone under certain age) or trigger based (e.g. low account balance).
  - **Product** is a fundamental element of the campaign. The aim of the product entity is to ensure that the business product conditions are not violated (especially necessary among the credit offers).
  - **Segment** entity is purely related to the business segments defined by the bank. It prevents generating irrelevant offers (e.g. targeting private clients with the basic credit loans). The other ability of this entity is for example to specify whether employees should be excluded or not.

- **Channel** entity characterizes the physical outbound channels with some minor exceptions such as distinguishing between compulsory (e.g. legal) offers and the others. The compulsory offers for example don't respect the marketing contact allowance provided by the client, whereas the product offers would violate legal restrictions if they were sent to clients who did not agree to be contacted with promotional offers. The channel base should be also divided to Primary (Push) and Secondary (Pull) channels. One campaign is assigned generally to one primary channel and up to a few secondary channels.
  - **Primary Channel** previously also referred as Push Channel directly targets the client with an offer, possible channels are Call center, E-mail, SMS, Mail etc.
  - **Secondary Channel** or Pull Channel reaches the client indirectly, example channels are Internet and Mobile Banking, ATM, Social networks and Search engines.

Market Overview

Figure 3.5: Magic Quadrant for Multichannel Campaign Management, April 2017, Source: (Gartner, 2017)

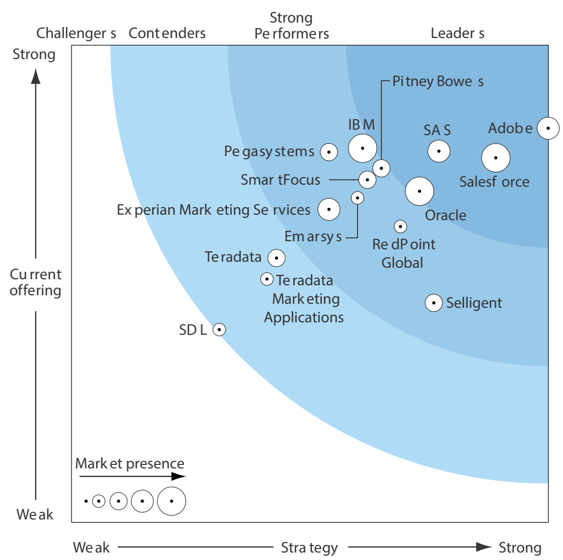

Figure 3.6: The Forrester Wave™: Cross-Channel Campaign Management, Q2 2016, Source: (Forrester, 2016)

Figures 3.5 & 3.6 represent the findings of the world's leading research and advisory companies Gartner and Forrester. Both companies refer Campaign Management Systems with a slightly different terminology – Multichannel Campaign Management in the Gartner's research and Cross-Channel Campaign Management in the Forrester's report.

Both companies emphasize the campaign management system vendor's ability of multi-channel campaign orchestration and real-time integration. Both research methodologies also evaluate statistical and machine learning capabilities to optimize the campaign placement, targeting and design (e.g. A/B testing).

Interestingly only little attention is paid towards customer lifecycle management or customer lifetime value. Despite the fact, that the customer related metrics are not directly incorporated into the evaluation methodology, (Forrester, 2016) points out the customer life-cycle focus of following vendors: SmartFocus[2], Emarsys[3] and Experian Marketing Services[4], (Gartner, 2017) mentions the niche players such as Listrak[5] or Pitney Bowes[6]. The low significance of customer metrics may be explained by the orientation of both researches on the retail industry. However, the visible trend of relatively new, niche players focusing on the problematic of customer evaluation can be identified.

Adobe[7] with its Adobe Campaign is a clear market leader in the both researches. According to (Forrester, 2016) customers describe the product as a real multichannel tool, with a lot of built-in functionality. The Adobe Marketing Cloud integrating services such as Adobe Analytics and Experience manager provides user-friendly statistical insights. The other strength mentioned by (Gartner, 2017) is the added value of Adobe Creative Cloud – the leading solution among graphical suites.

The second highest scoring vendor is SAS[8] with its SAS Customer Intelligence (CI). The differentiating factor is the most advanced marketing analytics on the market. For

[2] SmartFocus (The Message Cloud), http://smartfocus.com

[3] Emarsys (B2C Marketing Cloud), http://emarsys.com

[4] Experian Marketing Services (Experian Marketing Suite), http://experian.com/marketing-services

[5] Listrak (Digital Marketing Automation Platform), http://listrak.com

[6] Pitney Bowes (Commerce Cloud), http://pitneybowes.com/commercecloud

[7] Adobe (Adobe Marketing Cloud), http://adobe.com/marketing-cloud

[8] SAS (SAS Customer Intelligence), http://sas.com/customer-intelligence

this specific capability, SAS CI established a strong position mainly in financial services and telecommunications. SAS does not use CLV metric as the key driver of campaigns, but CLV is used for the client base clustering and more precise scoring – e.g. among propensity to buy models. (Forrester, 2016)

The next follower is Salesforce[9] with its strength in the SaaS-based marketing with an email backbone. Users highlighted its ease of use, real-time integration and multichannel orchestration. This solution is particularly useful for smaller and medium organizations (users criticized the pricing strategy) in the retail industry, integration with in-depth analytics and machine learning is limited. (Gartner, 2017)

3.4 Campaign Optimization

Not all marketing campaigns created by the campaign engines are suitable for execution. Therefore, follow-up steps to filter out and score campaigns are necessary. The targeting process has described (Katsov, 2018) as a 3-round selection, containing (1) hard targeting, (2) soft targeting and finally (3) thresholding, as it can be seen in the figure 3.7.

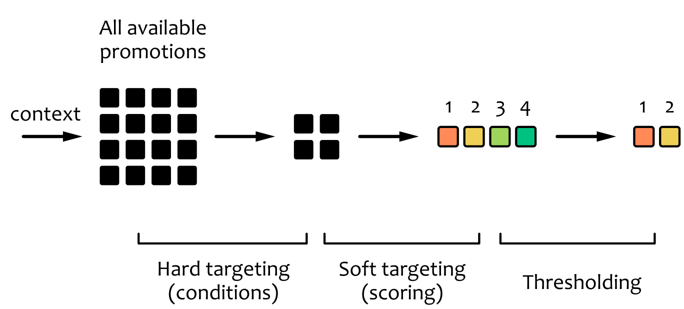

Figure 3.7: The campaign targeting process, Source: (Katsov, 2018)

The hard targeting (conditions) can be described as a process when campaigns violating some of the business rules are removed. The typical use case can be a campaign hierarchy, which defines the importance of one campaign compared to another. For example, credit card

[9] Salesforce (Salesforce Marketing Cloud), http://salesforce.com/marketing-cloud

campaigns hierarchy may be following – gold credit card is preferred to silver credit card and silver credit card to bronze one. Assuming the customer's eligibility for all these products, only the gold credit card campaign should be considered. Alternatively, some of the contact policy constraints may apply in this step. The contact policy may prevent addressing the customer to often or more importantly with campaigns which were already sent and refused by the client.

Whereas the Hard targeting (conditions) step is only diminishing the number of eligible offers, Soft targeting (scoring) & Thresholding represent the core of campaign optimization. Choosing the optimal subset of campaigns reflecting available budget and channel capacities is the aim of this process.

Some of the marketing channels have fixed capacities and their scalability is quite limited (typically call centers), while the digital channels (e.g. e-mails, SMSs, or online adds) have almost infinite capacity and their costs allow companies to send high amounts of offers over these channels. However, the tradeoff between digital channels and traditional ones can be usually found in their price and conversion rate. The lower conversion rate of digital channels might be caused by their ubiquity as for instance implementation of non-personalized email campaigns is straightforward.

On the other hand, channels such as call centers or postal mail services need significant initial investments. Moreover, their operation is interconnected with certain fixed costs, especially in terms of call centers. Employee's salaries and the depreciation of assets usually force companies to fully utilize these resources, even though more suitable channels could be used. Sometimes it is even difficult to predict channel capacities – for example in case of sudden service shortage the call center may be fully utilized only by the inbound traffic of customer complaints. A solution could be found in outsourcing of these services to external subjects. Despite the benefits of this solution, a potential problem can be identified in the quality assurance.

The scoring and thresholding algorithms must meet the business specific conditions and can be developed as either short term oriented (proposing the next best action only) or long term oriented (considering the customer lifecycle and lifetime value) and therefore building the individual marketing strategy.

Short-term oriented optimization algorithms (next best action)

Companies (Forrester, 2016) and (Gartner, 2017) mentioned in their researches the capabilities of different vendors with different solutions to optimize the targeting strategy. Generally, two main approaches can be identified: (1) maximization of the possible outcome (a propensity to buy/acquire a product multiplied by the selected metric - e.g. Profit, Revenue) and (2) the more common form is maximization of Return on Investment (ROI) as described in the equation 3.3.

$$ROI = \frac{(p \times \pi) - c}{c} \tag{3.3}$$

Where

- p = propensity to buy/acquire a product
- π = potential gain (e.g. Profit) from the product for the given customer
- c = the sum of costs of all used channel per campaign

Both approaches have its positives and negatives. The key problem of the approach maximizing ROI lays in the low utilization of costly channels. This issue can be described on the following situation (equation 3.4).

$$\frac{p_{pc} \times \pi - c_{pc}}{c_{pc}} = \frac{p_e \times \pi - c_e}{c_e} \rightarrow \frac{c_{pc}}{p_{pc}} = \frac{c_e}{p_e} \rightarrow p_{pc} = \frac{c_{pc}}{c_e} \times p_e \tag{3.4}$$

Let's assume that a product can be offered via 2 channels (phone call pc, and email e). If the potential gain π remains the same, the variables that differ are costs c and propensities p. In order to achieve the same ROI, the propensity of call center must be equal to the ratio c_{pc}/c_e multiplied by the propensity of email campaign. If a phone call is 10 times more expensive than an email, the propensity of the call center campaign must be 10 times higher, which is quite rare. To deal with this issue, propensity or cost adjustments may be applied.

Not surprisingly such behavior violates the rational investors premise to maximize the ROI and leads to the non-optimal solution.

The approach based on maximization of the possible outcome is sometimes applied in product driven campaigns. The typical situation, when this scenario is applied, occurs if a new product is launched and the key metric is to acquire a certain number of new clients. Top n propensity deciles are selected, and the campaign is executed regardless channel costs.

Even though both approaches are undoubtedly more efficient than targeting random customers, they can be described as semi optimal. They cannot be referred as optimal as there is no relationship between the marketing action and customer's needs related to the customer lifecycle phase. For instance, higher risk of customer's churn belongs to consequences of inappropriate marketing actions.

Long-term oriented optimization algorithms (individual marketing strategy)

While the aim of short-term optimization algorithms is to pick the next best action only, long-term oriented optimization algorithms have to reflect customer life-cycle phases or customer states, their dynamics and must be in line with the business strategy. For this purpose, Customer Lifetime Value (CLV) seems to be the most suitable metric. Despite the fact that CLV should reflect changes in customer attitudes, it may not fully reveal how a customer will behave in the future. Therefore, in some cases it is worth choosing campaigns with a negative impact on the target metric (CLV in this case), if the customer dynamics forecasts a potential gain in the future. This behavior is solely a feature of long-term optimization algorithms.

Modeling of CLV, customer dynamics and marketing strategy reflecting the most optimal marketing actions is discussed in the next chapter.

Summary

The key principles of retail banking and customer targeting were discussed in the chapter. A marketing campaign framework reflecting industry specifics was introduced. Last but not least, the main benefits of transformation from short-term oriented to long-term, customer centric marketing approach were mentioned.

Chapter 4

CLV prediction and optimization

The aim of this chapter is firstly to summarize models used for CLV prediction in the banking sector, secondly to propose an algorithm capturing customer dynamics and enhancing CLV via marketing actions.

4.1 Predictive models for CLV

Even though key benefits of the CLV metric for customer performance management were mentioned in previous chapters and thorough description of predictive models used for the CLV modeling is a great part of the literature review, models and variables specifically used in a retail banking context were not yet discussed.

An extensive research focused on CLV methodologies and variables evaluation conducted (Ekinci et al., 2012). Their findings transformed into a chart are visualized in the figure 4.1. It is important to mention that as multiple methodologies could have been used in a single study, the total percentage (the sum of percentages of all methodologies mentioned in the figure) does not sum up to 100%.

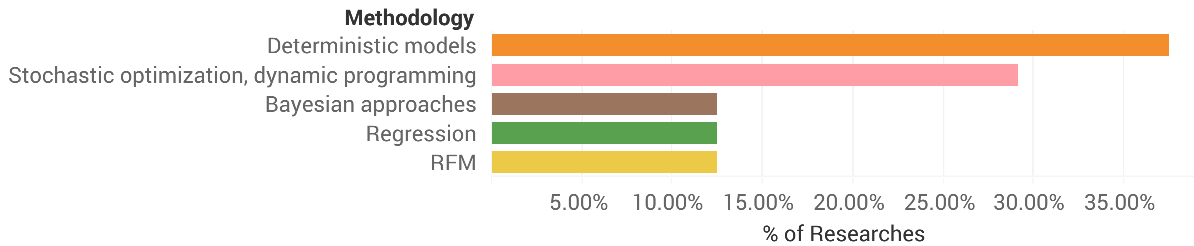

Figure 4.1: Methodologies used to model CLV within the banking industry, underlying research (Ekinci et al., 2012), Source: Author

Deterministic models

From the figure 4.1 is clearly visible the trend of simplification of modeling approaches to the deterministic methodologies. Such methodologies suggest mostly discounting of current profits, while expecting an average customer lifespan to estimate the length of time-series. Deterministic approaches are well summarized in the research conducted by (Ferrentino, Cuomo, & Boniello, 2016), some of the calculation formulas contain acquisition costs, contribution margin, or profit function. However, limitations found by (Ekinci et al., 2012) are for example: *profit margin is taken as fixed*, or *customer is assumed to buy or drive a car until his death*. These approaches can be used for the descriptive analyses of a current customer portfolio, but their benefits for future estimates are limited or even misleading.

Stochastic optimization, dynamic programming

The second group of the most relevant methodologies is stochastic optimization and dynamic programming. These approaches are mostly based on the theory of Markov chain and Markov Decision process. The key advantage of these models is their visual interpretability with various forms of Sankey diagrams or theory of graphs. The possibility to measure a natural customer's behavior and effects of marketing campaigns, makes from this group of algorithms an ideal tool for calculation and optimization of CLV. Plus, the theory of Markov Decision process with its roots in 1960s is well developed and several types of optimization algorithms are implemented in modern data science languages such as R, Python or Matlab. However, as identified in (Ekinci et al., 2012) the greatest challenge remains to correctly determine customer states and transition probabilities. Markov chains and Markov Decision process is further elaborated in the separated part of this chapter.

Bayesian approaches

Models based on Bayes' theorem (Pareto-NBD, hierarchical Bayes approach, and Bayesian decision theory) are ideal in situation when the decision must be made with substantial parameter or modeling uncertainty. Bayesian decision theory defines 3 components: actions (marketing actions per individual customer), states (quantity and timing of purchases by customers), and consequences (e.g. profits created by combinations of actions and states). Bayes' theorem is applied in prediction of future customer states based on prior distribution to reduce the uncertainty. The optimal action maximizes the expected outcome (e.g. profit), which is computed with respect to the predictive distribution of future states (i.e. quantity and timing). (Venkatesan, Kumar, & Bohling, 2007)

Algorithms using Bayesian approaches for CLV prediction are extensively dependent on correct estimation of the underlying distributions. In specific cases the distribution can be directly observed from the transactional data (e.g. Poisson distribution of new cash loans, or Gamma distribution of terminating subscription services). However, data may include noise or hidden patterns and therefore in most of cases it is difficult to predict the customer states.

Regression models

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n + \epsilon \tag{4.1}$$

Linear regression models estimate the value of dependent variable (y) given the vector of independent variables (X), a calculation formula is expressed by the equation 4.1. Methods such as Least Square Error (LSE), Maximum Likelihood Estimation (MLE), or Artificial Neural Network (ANN) are used to estimate the coefficients $\beta_1, \beta_2, \ldots, \beta_n$, while minimizing the value of ϵ (error). Although regression models generally do not belong to the best performing models in terms of value prediction, important business impacts may be found in the β coefficient estimates, i.e. whether the given predictor increases or decreases the target variable and to what degree.

Methodology proposed by (Ekinci et al., 2012) contains of 2 steps: firstly, CLV calculation depending on empirical data stored in the database and secondly modeling the target variable (CLV) based on a vector of independent variables valid in a single period. The drawback of this methodology lays in the length of forecast period – the shorter the timeframe is, more precise results can be obtained. On the contrary especially in a banking sector at which the study was aimed, the customer lifecycle is much longer than one year suggested by the authors. Therefore, the calculated CLV as a sum of discounted profits within one year can be regarded as only a fraction of the actual CLV. Nevertheless, as the calculation is consistent throughout the customer portfolio, it may be sufficient for further linear regression modeling. Authors used two methods (LSE & ANN) to obtain β coefficients from which the LSE was selected due to higher performance.

Even though (Ekinci et al., 2012) used an inappropriate CLV calculation method, their research is particularly useful in terms of predictor's evaluation. Based on the statistical significance and an exploratory study they created a list of important variables for CLV prediction. The list includes following attributes: product related variables (types of products, number of products, new product usage behavior); profitability of the customer; monetary values (total assets, values of the specific services used by the customer); monetary risk (risk of default of loan or credit card payments); activity level of the customer; total value of the salary payments.

RFM models

Recency-frequency-monetary (RFM) models were already described in the literature review. The most important feature of such models is their great performance in CLV prediction in industries, which do not store or dispose detailed client information. Studies

(Venkatesan & Kumar, 2004; Keiningham et al., 2006; Gupta et al., 2006) have successfully used these models. However, in terms of retail banking much more detailed information about customers than just recency, frequency or monetary value of purchases is available and therefore these models are rather used as benchmarks.

4.2 Modeling customer dynamics and actions enhancing CLV

The concept of modeling customer dynamics can be found in a broad range of studies and researches, (Pfeifer & Carraway, 2000; Peter Paauwe, 2007; Cheng et al., 2012; Mzoughia & Limam, 2014). The authors generally choose Markov Chain (MC) or more complex Markov Decision Process (MDP) to reveal the relationships between customer epochs and predict customer behavior in the future.

Whereas Markov Chain (MC) models are rather descriptive as they only capture the transition probabilities between customer states and therefore are ideal for CLV prediction, Markov Decision Process (MDP) models can be described as stochastic optimization algorithms as they allow to handle not only natural transition probabilities but set of possible actions (e.g. marketing campaigns) and possible outcomes, and choose the optimal strategy to target the customer. However as both approaches share the essential components, MC will be discuseed first and MDP later in this chapter.

4.2.1 Markov Chain (MC) model

> **Markov Chain (MC)** is a tuple $\langle S, P, R \rangle$ in which S is a finite set of states, P a transition function defined as $P : S \times S \rightarrow [0, 1]$ and R a reward function defined as $R : S \times S \rightarrow \mathbb{R}$. (van Otterlo & Wiering, 2012)

Graphical representation of the above stated definition is shown in the figure 4.2. The figure captures a 4-state (S) Markov Chain model, links between states represent both transition probabilities and rewards of shifts between $S \rightarrow S'$. The grey arrows distinguish a specific case – a transition to the same state and therefore the reward is assumed to be equal to 0. Additionally, the state S_0 can be regarded as a terminate state as churned customers are assumed to stay inactive forever. Note that there is no difference between arrow colors, the aim it to visually distinguish those connections.

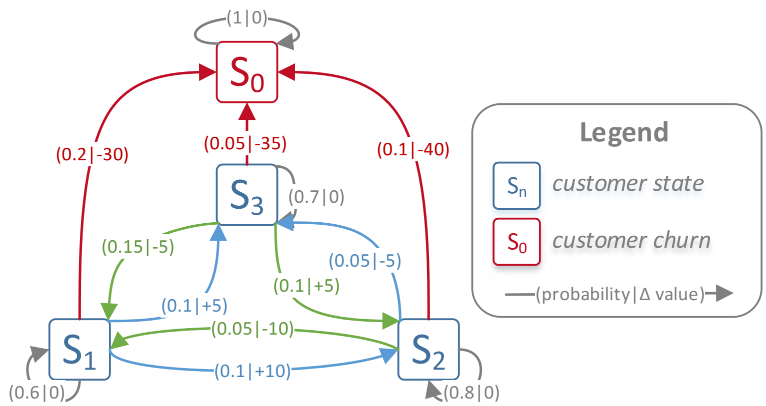

Figure 4.2: A Markov chain specifying a customer migration based model, Source: Author

Even though the definition and the explanatory figure 4.2 may be sufficient for the understanding of MC models, key MC components and assumptions are discussed further.

Markov Chain components

- **States:** The set of environmental states S is defined as the finite set $\{s^1, ..., s^N\}$ where the *size* of the state space is N, i.e. $|S| = N$. A state is a unique characterization of all that is important in a state of the problem that is modeled. (van Otterlo & Wiering, 2012)

  In other words, all states must be externally heterogenous (distinguishable from each other) and internally homogenous (should be describing only one particular situation or set of situations that are mutually replaceable). From the mathematical perspective there is no limit in the number of states, in extreme case one state for every single customer can be observed. Even though such behavior matches the above stated definition, one can assume, that this is not the most practical solution. Not only transition probabilities between states could be hardly measured in that case, having millions of states would lead to extreme resource requirements (memory and computation power). Based on author's experience, the sufficient number of states in the banking environment fluctuates in range of 50 to 150, depending on the size of customer base and product portfolio variety. A state then represents a cluster of clients with similar behavioral pattern and business needs. The customer state is further also referred as a customer microsegment.

- **Transition matrix:** Once the states are defined, transition probabilities can be calculated from the historical data by aggregating the customer's flow. Transition matrix is an array of $N \times N$ transition probabilities where rows represent the customer state in the period t_0 and columns the customer state in period t_1, the values then represent the probability of changing state $S \to S'$ between periods t_0 and t_1. The sum of probabilities in each row must be equal to 1, whereas the column sum may differ. A mathematical form of the transition matrix is shown in the equation 4.2.

$$\mathbf{P}(S,S') = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ \vdots & \ddots & \vdots \\ p_{n,1} & \cdots & p_{n,n} \end{bmatrix} \tag{4.2}$$

- **Reward:** Sometimes also referred as a reward function, is represented by a N-size vector R of values related to customer states $r(s)$. As visualized in the figure 4.2 the reward may depend on the previous customer state $r(s' \mid s)$. In such cases as discussed in (Tirenni, 2005), the calculation is as follows: $r(s) = \sum_{s' \in S} p(s' \mid s) \times r(s' \mid s)$. Financial metrics (e.g. revenue, profit) are usually used as reward functions.

Markov Chain properties

- **Markov property:** The idea behind Markov property is that the transition probabilities are affected only by the current state, i.e. knowing the customer's state is sufficient to predict the future behavior. Hence no historical impact on the current state is assumed. However, cases when the probabilities are dependent on k-last states exist. An example could be retention activities – one can assume that if the retention process is successful and customer is acquired back, he or she is most likely to return to a customer state same or similar to the one before leaving. The solution is to create as many "churned" states as necessary to distinguish previous customer states (e.g. instead of one churn state, there could be high-profit churn, medium-profit churn and low-profit churn). In general, every k-Markov problem can be transformed into an equivalent Markov problem (1-Markov problem), where k represents the number of states impacting the current transition probabilities.

- **Stationary & Non-stationary Markov chains:** States are considered to be stationary in all types of Markov chains. On the other hand rewards and transition probabilities may evolve over time. Even though techniques to handle non-stationary dynamics exist, the modeling exercise is far more complex and usually does not pay off. Due to this fact, MC models with fixed transition matrix and reward vector are used commonly for CLV modeling. (Tirenni, 2005; van Otterlo & Wiering, 2012)

Customer Lifetime Value prediction

When the states, rewards and transition probabilities are determined, CLV of each customer state can be calculated using the equation 4.3.

$$CLV(s_n) = \left[\sum_{t=0}^{T} \mathbf{I} \times \mathbf{P}^t \times \mathbf{R} \times \frac{1}{(1+i)^t}\right]_n \tag{4.3}$$

Where

– T = time horizon, eihter limited by business requirements or infinite as the value converges (is explained further)

– $\mathbf{I}$ = identity matrix of size N (number of states)

– $\mathbf{P}$ = transition matrix

– $\mathbf{R}$ = reward vector

– i = discount rate

To estimate CLV for the whole customer base, it is necessary to obtain a vector of customer counts in each state and multiply it by the vector of CLVs per customer state forecasted by Markov chain model, as mentioned in the equation 4.4.

$$CLV = CLV(s) \times \mathbf{M} \tag{4.4}$$

Where $\mathbf{M}$ is a vector of current customer counts in each state.

Python implementation of the CLV calculation algorithm is listed in the appendix A.

Markov Chain example

The above mentioned Python algorithm can be used for CLV simulations. The impact of different time horizon and discount rate while having fixed transition matrix and reward vector on CLV value is shown in the figure 4.3. The transition matrix and reward vector used for the simulation are mentioned in the equation 4.5.

$$\mathbf{P} = \begin{bmatrix} 0.80 & 0.05 & 0.05 & 0.10 \\ 0.01 & 0.95 & 0.02 & 0.02 \\ 0.01 & 0.04 & 0.90 & 0.05 \\ 0.03 & 0.03 & 0.03 & 0.91 \end{bmatrix} \mathbf{R} = \begin{bmatrix} 300 \\ 500 \\ 800 \\ 1000 \end{bmatrix} \tag{4.5}$$

Note that the above stated example does not include the terminate state (customer churn). The intention is to rather show the influence of discount rate and time horizon than simulate a real-world case.

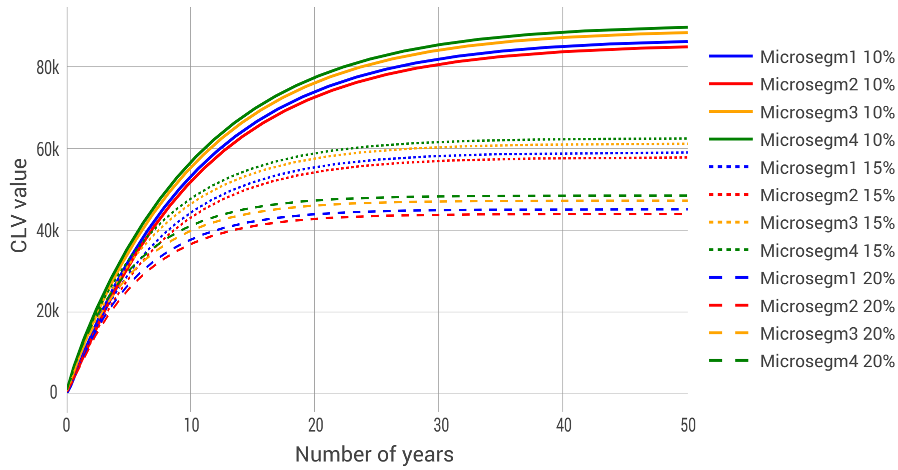

Figure 4.3: Relationship between CLV value, selected time-frame and discount rate, visualized for 4 different customer states (microsegments), Source: Author

In the figure 4.3 is visualized the relationship between a selected time horizon and CLV value of the customer state. Furthermore 3 different levels of discount rate (namely 10%, 15%, and 20%) are displayed to provide an insight of how the discount rate impacts the overall CLV value. The plot shows that the CLV value converges at certain point in the future which is affected by selected discount rate. What higher the discount rate is, the shorter time horizon is needed for CLV value to converge.

4.2.2 Markov Decision Process (MDP) model

The aim of this section is firstly to introduce Markov Decision Process and secondly propose a concept of applying MDP algorithms to optimize marketing campaign strategy. Additionally, to the Markov Chain model of customer states and transition probabilities, MDP defines a set of actions which are used to maximize obtained rewards over time. Such maximization is achieved by various techniques of dynamic programming which are described at the very end.

Markov Decision Process (MDP) is a tuple $\langle S, A, P, R\rangle$ in which S is a finite set of states, A a finite set of actions, P a transition function defined as $P : S \times A \times S \rightarrow [0, 1]$ and R a reward function defined as $R : S \times A \times S \rightarrow \mathbb{R}$. (van Otterlo & Wiering, 2012)

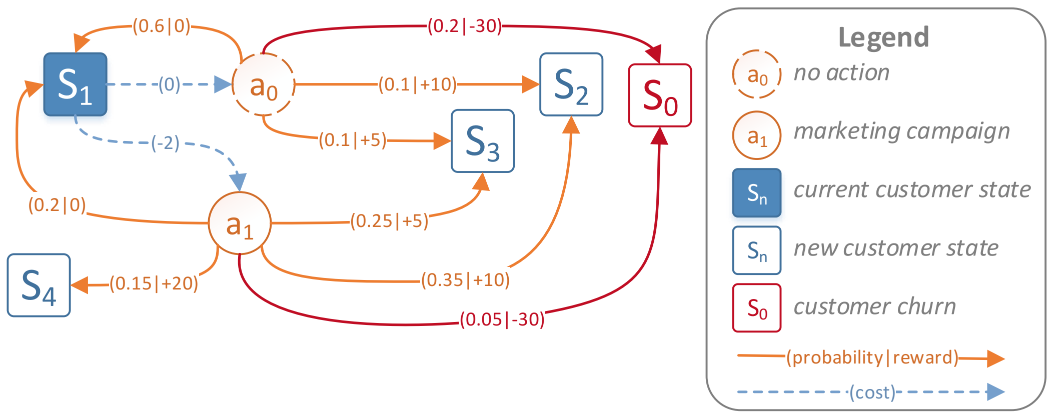

Figure 4.4: Example of modeling customer dynamics with Markov Decision Process, Source: Author

The figure 4.4 displays a 5-state MDP with just two actions. For the sake of clarity, only relations to the state S_1 are displayed. Action a_0 represents "no-action", i.e. this link shows natural customer's behavior if no marketing campaign (action) is launched. MDP with only a_0 relationships can be derived from the previous MC model, as all the necessary information to build such model are already known (transition probability $S \rightarrow S'$, reward $r(s)$, and cost of no-action always equals zero). However, to model MDP, a transition matrix for each marketing campaign (action) must be estimated and its cost measured. The corresponding reward is obtained then from the transition probabilities and state values similarly to the concept used in MC model.

Markov Decision Process components

- **State:** The finite set of states $\{s_1, \ldots, s_N\}$ defines the MDP environment. Size of the state space is N, i.e. $|S| = N$. A state is a unique characterization of all that is important in a state of the problem that is modeled (van Otterlo & Wiering, 2012). The discussion about state properties was already mentioned in the MC section – as the state implications are identical, please refer to MC model for clarification.

- **Action:** An action is an essential component that distinguishes MDP models from MC ones. The set of actions A is defined as the finite set $\{a_0, \ldots, a_{K-1}\}$ where the size of the action space is K, i.e. $|A| = K$. Actions are used to control the system state. Every action interconnects two states – either identical $s \rightarrow a \rightarrow s$ or different $s \rightarrow a \rightarrow s'$. The link is described with its probability, cost and finally reward. Two types of actions exist in MDP modeling. Firstly, "no-action" a_0 representing the natural flow of customers between customer states without being influenced by marketing campaigns or any other activities. It is necessary to mention that zero costs are related to "no-action" activities. Whereas marketing incentives acting as the second group of actions are always related with specific costs and may have positive, zero or even negative impact on the customer value. Each type of marketing campaign results in at least one action in the MDP model. Multiple instances of the same type of marketing campaign can be used, e.g. different costs of marketing campaigns depending on the initial customer state. However, such nuances are usually already incorporated in the campaign management system and result in different campaigns. Therefore, they are treated exclusively by MDP models. Due to this fact, fixed action costs per customer states are assumed.

- **Transition function:** The transition function defines a probability $p(s'|s, a)$, i.e. the probability of resulting in s' in period $t_{period+1}$ while being in period t_{period} in state s and applying action a. Therefore, one transition matrix of size $N \times N$ as used in MC models is not sufficient. Instead, every action must be represented by a corresponding transition matrix. However, the concept is similar to MC model as rows represent the initial state and columns the following. Unlike in MC models, the sum of transition matrix rows in MDP models can be either equal to zero (which implies that the action is not defined in the initial customer state) or equal to one as in MC models.

- **Reward function:** The reward function specifies rewards for being in a state s, taking an action a and resulting in a state s', therefore $R : S \times A \times S \to \mathbb{R}$. The reward dependent on the initial and ending state, and an action taken $r(s, a, s')$ can be transformed into a reward depending solely on the initial state and an action $r(s, a)$ with following equation as discussed in (Tirenni, 2005).

$$r(s, a) = \sum_{s' \in S} p(s'|s, a) \times r(s, s') - c(a) \tag{4.6}$$

  Where $p(s'|s, a)$ stands for the transition probability and $C(a)$ for action costs. The reward function is then represented by a matrix of size $N \times K$, i.e. defining a reward for every single customer state and every possible action. The reward is a crucial part of MDP models as it implicitly specifies the goal of learning (van Otterlo & Wiering, 2012).

- **Decision epoch:** Each time period is called a decision epoch as the term refers to the dynamic character of the MDP model. In every decision epoch $\{t_0, \ldots, t_n\}$ a decision, which action to perform with respect to a state, must be done.

Markov Decision Process properties

- **Policy:** A policy π is a function that defines for a given MDP $\langle S, A, P, R \rangle$ for each $s \in S$ an action $a \in A$, i.e. $\pi : S \to A$ (van Otterlo & Wiering, 2012). Therefore the policy is an N-size vector of action a references: $\pi = \{\pi_1, \ldots \pi_N\}$. The aim of MDP algorithms is to compute an optimal policy π^* which maximizes a value function.

- **Value function:** A value function indicates *how good*, i.e. in terms of the expected return, it is to be in a certain state, or to perform a certain action in a specified state. The value of a state s under policy π, denoted $V^\pi(s)$ is the expected return when starting in state s and following policy π thereafter (van Otterlo & Wiering, 2012). An equation 4.7 expresses a discounted infinite-horizon value calculation.

$$V^\pi(s) = E_\pi \left\{ \sum_{k=0}^{\infty} \gamma^k \times r_{t+k} \mid s_t = s \right\} \tag{4.7}$$

Where γ represents the discount factor.

- **State-action value function:** Some of the optimization algorithms consider the state-action value function $Q : S \times A \to \mathbb{R}$, defined similarly to state value function $V^{\pi}(s)$ as:

$$Q^{\pi}(s,a) = E_{\pi}\left\{\sum_{k=0}^{\infty} \gamma^{k} \times r_{t+k} \mid s_t = s, a_t = a\right\} \tag{4.8}$$

- **Bellman equation:** The fundamental recursive property the value function calculation defined (Bellman, 1957) as:

$$V^{\pi}(s) = \sum_{s'} P(s,a,s') \times \Big(R(s,a,s') + \gamma V^{\pi}(s')\Big) \tag{4.9}$$

In other words, the Bellman equation defines an expected value of a state as the immediate reward and values of possible next states weighted by their transition probabilities, discounted by the discount factor γ. Multiple policies may end up with the same value function, but for a given policy π, the V^{π} is unique.

The aim of MDP algorithms is to find the optimal policy, i.e. the policy that maximizes an obtained reward. The best policy then maximizes the value function defined in equation 4.7 for all states $s \in S$. An optimal policy π^{*}, is such that $V^{p}i^{*}(s) \geq V^{\pi}(s)$ for all $s \in S$ and all policies π. Equation 4.10 describes the value function of optimal policy.

$$V^{*}(s) = \underset{a \in A}{max} \sum_{s' \in S} P(s,a,s') \Big(R(s,a,s') + \gamma V^{\pi}(s')\Big) \tag{4.10}$$

Markov Decision Process optimization algorithms

Algorithms used for MDP optimization can be divided into two main classes: (1) Dynamic Programming (DP) algorithms and (2) Reinforcement Learning (RL) algorithms. Whereas DP algorithms can compute optimal policies only in the presence of a perfect model of the environment, RL algorithms may deal with some level of uncertainty. In many applications it is hard to ensure the availability of a perfect model, therefore the DP models hit their limits. However, from theoretical viewpoint, DP algorithms set fundamental computational mechanisms which are then further used in RL models (van Otterlo & Wiering, 2012).

As the MDP model in terms of CLV optimization can be regarded as a perfect model (all the components – states, actions, transition function, and finally reward function are known), both key DP algorithms, namely Policy iteration and Value iteration will be introduced. As a benchmark RL algorithm Q-Learning, which is available in multiple MDP libraries, is discussed[1].

- **Policy Iteration:** Policy iteration algorithm proposed by (Howard, 1960) consists of two interaction processes as shown in figure 4.5. Firstly, *policy evaluation* step estimates the value function (utility) of the current policy π. Secondly, the *policy improvement* phase computes an improved policy by maximization over the value function. The improved policy is computed greedily by selecting the best action with respect to the value function. Stopping condition is met when the process converges to an optimal policy.

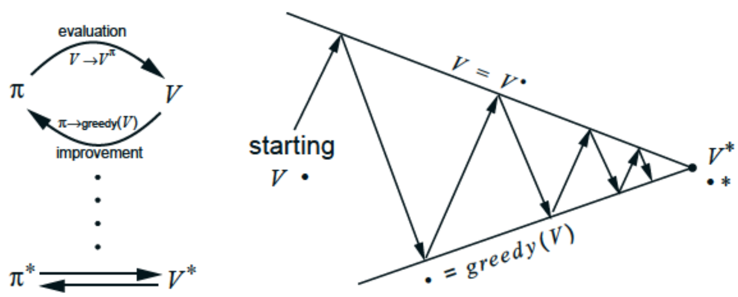

Figure 4.5: The gradual convergence of both the value function and the policy to optimal versions, Source: (van Otterlo & Wiering, 2012)

[1]A comprehensive overview of MDP algorithms can be found in (van Otterlo & Wiering, 2012).

Require: $V(s) \in \mathbb{R}$ and $\pi(s) \in A(s)$ arbitrarily for all $s \in S$
{POLICY EVALUATION}
repeat
$\Delta := 0$
for each $s \in S$ **do**
$v := V^{\pi}(s)$
$V(s) := \sum_{s'} T(s, \pi(s), s')\Big(R(s, \pi(s), s') + \gamma V(s')\Big)$
$\Delta := \max(\Delta, |v - V(s)|)$
until $\Delta < \sigma$
{POLICY IMPROVEMENT}
policy-stable := true
for each $s \in S$ **do**
$b := \pi(s)$
$\pi(s) := \arg\max_a \sum_{s'} T(s, a, s')\Big(R(s, a, s') + \gamma \cdot V(s')\Big)$
if $b \neq \pi(s)$ **then** policy-stable := false
if policy-stable **then** stop; **else** go to POLICY EVALUATION

Algorithm 1: Policy Iteration, Source: (van Otterlo & Wiering, 2012)

- **Value Iteration:** While Policy Iteration algorithm completely separates policy evaluation and policy improvement phases, the value iteration algorithm does not wait for a full convergence and evaluates the policy just after one iteration and improves the policy based on the evaluation so far. The algorithm was originally proposed by (Bellman, 1957), the main objective is to maximize the value function V^* using the equation 4.9.

Require: initialize V arbitrarily (e.g. $V(s) := 0, \forall s \in S$)
repeat
$\Delta := 0$
for each $s \in S$ **do**
$v := V(s)$
for each $a \in A(s)$ **do**
$Q(s,a) := \sum_{s'} T(s, a, s')\Big(R(s, a, s') + \gamma V(s')\Big)$
$V(s) := \max_a Q(s,a)$
$\Delta := \max(\Delta, |v - V(s)|)$
until $\Delta < \sigma$

Algorithm 2: Value Iteration, Source: (van Otterlo & Wiering, 2012)

- **Q-Learning:** The Q-Learning algorithm belongs to family of RL algorithms. The basic idea in Q-learning is to estimate Q-value (state-action value function) by exploring the state space, while choosing different actions and getting feedback (reward). The algorithm saves obtained Q-values and selects the maximal Q-value in order to update

Q_t into Q_{t+1}. The terminating condition can be either ending in the desired state or passing a certain number of iterations.

Require: discount factor γ, learning parameter α
initialize Q arbitrarily (e.g. $Q(s,a) = 0, \forall s \in S, \forall a \in A$)
for each episode **do**
s is initialized as the *starting* state
repeat
choose an action $a \in A(s)$ based on an exploration strategy
perform action a
observe the new state s' and received reward r
$Q(s,a) := Q(s,a) + \alpha \left(r + \gamma \cdot \max_{a' \in A(s')} Q(s',a') - Q(s,a) \right)$
$s := s'$
until s' is a goal state

Algorithm 3: Q-Learning, Source: (van Otterlo & Wiering, 2012)

Summary

The proposed Markov Chain model captures customer dynamics and calculates CLV based on the customer's behavior. The model is particularly useful in industries with a solid data background such as banking or telecommunications. On the other hand, segments of economy where the customer's history is unknown should use RFM, Regression or Bayesian approach-based models which showed of a great performance throughout researches.

Markov decision process represents a higher grade of CLV modeling as it estimates an optimal marketing strategy. While being consistent with the MC model, direct impact on the marketing performance can be measured. From the business perspective MDP model completely changes the way how marketing strategy is created. Turning focus from short-term optimization (e.g. profit oriented) to customer centric, CLV based model with appropriate targeting.

Chapter 5

Implementation of Markov decision process for CLV optimization

This chapter provides guidance how to model CLV using Markov Chain. Based on the proposed Markov chain model, the Markov Decision Process, which optimizes marketing campaigns regarding CLV and customer dynamics, is designed.

From the technology perspective all the methodology steps can be reproduced in various relational database management systems in cooperation with statistical software e.g. SAS, SPSS, RapidMiner. However, as large data volumes are being processed (millions of customers making thousands of transactions result in billions of data records), it is suggested to use distributed data storage and processing components which split the workload to multiple nodes. Therefore, they can provide results faster and their scalability, if needed, is greater. Apache Hadoop [1] and Apache Spark[2] were selected for their capability not only handle distributed data processing, but for the machine learning libraries already included in the Apache Spark engine and their close integration. Thus, there is no need to transfer data from one environment to another, which also significantly improves either the speed or volume of data possible to process. Furthermore, as of now Apache Spark (2.3.0) implements APIs to 4 different programming languages, namely: Scala, Java, Python and R. Deployment

[1] More information about Apache Hadoop can be found at: http://hadoop.apache.org

[2] Details about Apache Spark are available at: https://spark.apache.org/

of the proposed solution in the current enterprise environment is hence simplified.

The proposed component architecture is shown in the figure 5.1. Data unification and campaign prioritization components are not included in the methodology for their high dependency on the target environment.

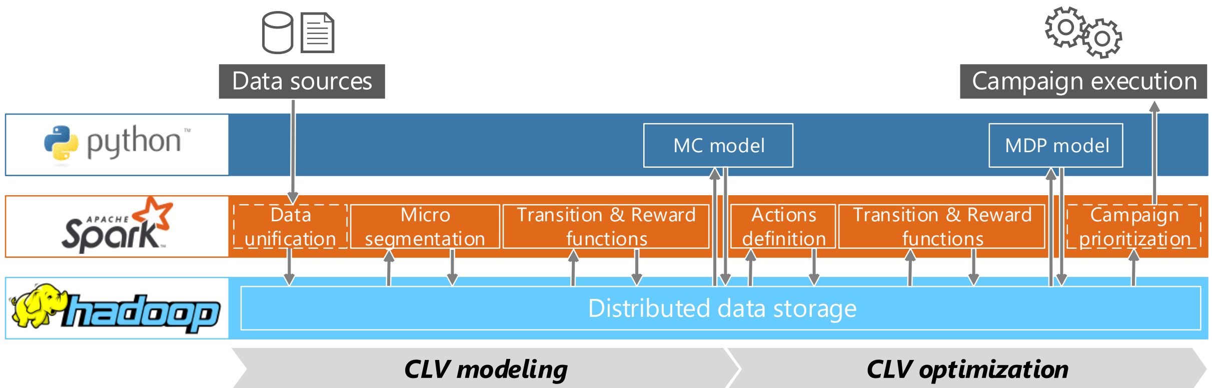

Figure 5.1: CLV optimization pipeline, technology and component overview, Source: Author

5.1 CLV modeling

Various CLV predicting algorithms were mentioned in chapters 2 & 4, from which Markov Chain model is the most suitable for the purpose of CLV optimization. Two main reasons can be identified: (1) in environments with a rich data background, Markov Chain model performs very well, (2) consistent customer state definition among the CLV estimation and optimization algorithms diminishes the risk of potential errors and enhances the overall understandability of a whole solution. Finally, a clear graphical representation of model outputs, visualized in this section, adds Markov Chain model the desired credibility from business perspective.

5.1.1 Microsegmentation

Clustering the customer base into sufficiently small, internally homogeneous and externally heterogenous "microsegments" is the most challenging part of the Markov Chain model building process. As proposed by (Cheng et al., 2012), the microsegmentation framework should be based on three main pillars: (1) Lifetime Prediction (churn score), (2) Profit Prediction (profit or eventually revenue) and (3) Behavioral Prediction.

Churn prediction is an ordinary classification problem based on algorithms such as Logistic Regression, Decision Tree, Random Forrest, Support Vector Machines etc. while having features from customer profile, referred in chapter 3.3, as predictors and binary class (churned/not churned) as a target variable. An ensemble model to tune the final model stability and prediction performance can be used as discussed in chapter 2. Some authors e.g. (Chamberlain et al., 2017) used linear regression to calibrate the model outputs in order to precisely fit the observed churn probability. However, as the churn score will be later used only to split customer base to deciles, the scale does not matter. Predicting churn score is not the aim of the thesis and thorough methodology is therefore not provided.

Similarly, the second pillar (Profit Prediction) must be provided by the banking institution. The input solely depends on internal methodologies of asset evaluation. Even though cash flow from some product groups or services can be directly determined, e.g. fees related to account or payment card usage, the assessment of profits/losses from other kinds of product or service requires significant business knowledge, if even possible to exactly calculate. For example, loan profitability depends on the risk of customer's default and is therefore highly individual. Although CLV is defined as the sum of all future discounted profits (see chapter 3.2), sometimes it is not possible to forecast profits. In such cases the revenue can be used as an approximated solution.

Behavioral segmentation

Banks generally split their customer base into business segments, which are treated differently (e.g. private banking and mass market customers). Even though business segments are limited only to customers with certain income or savings volume, once the customer classifies for the desired business segment, his or her family relatives automatically obtain the same privilege. Due to this fact an enormous heterogeneity among business segments exists, therefore an additional behavioral segmentation is necessary.

On the contrary, behavioral segmentation is entirely based on customer characteristics stored in the customer profile (e.g. monthly income, number of transactions or product specific information such as the sum of money owed, invested etc.). As no target class exists, the segmentation can be described as an unsupervised learning task. The aim of this machine learning task is to find groups of clients which have similar behavioral patterns within the cluster and differ from the others.

One of the most used classification algorithms for such type of tasks is the K-means algorithm. Where K represents the number of clusters created by the algorithm. The algorithm initiates a set of K-centroids and iteratively changes their position to minimize the cost function ("Spark Documentation", 2018). The cost function is represented by Within Set Sum of Squared Errors (WSSE) and defined in equation 5.1.

$$WSSE = \sum_{n=1}^{N} \left\| x_n - center_k(x_n) \right\|^2 \tag{5.1}$$

Where

- N = number of observations (customers)
- x_n = n-th observation (characteristics of a particular customer)
- $center_k(x_n)$ = the closest centroid position to the observation x_n

What more clusters exist, more precisely the algorithm fits customer base and therefore WSSE decreases. The relationship between the number of clusters and WSSE is shown in the figure 5.2. As the limit of WSSE function equals zero (when number of clusters equals to number of customers) it is necessary to define a stopping criterion. From mathematical perspective the stopping criterion is defined as maximum of derivation of WSSE function, i.e. the relative minimal enhancement of the cost function when defining an additional cluster, as stated in equation 5.2.

$$\frac{WSSE_{k+1} - WSSE_k}{WSSE_k} \leq \delta \tag{5.2}$$

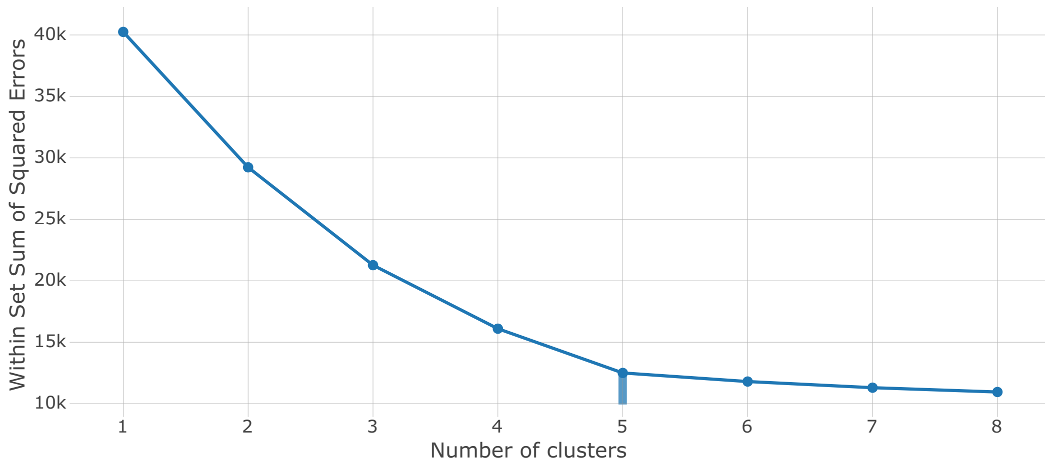

Figure 5.2: A K-means simulation, relationship between number of clusters and WSSE, Source: Author

The figure 5.2 shows the relationship between number of clusters and WSSE, with an optimal number of five clusters highlighted (based on the stopping condition mentioned in equation 5.2). However, from the business standpoint a different number of clusters can be meaningful and therefore preferred.

Profit and Churn bins

As referred by (Gupta et al., 2006; Cheng et al., 2012) profit and churn are key drivers of CLV. To consider both variables equally, the target segment is a combination of discretized profit and churn score. To obtain similarly sized variable bins, the array of variables must be sorted and then split. Sorting values is a costly operation as the data must be gathered from execution nodes and sorted on a single machine (usually the master node). To diminish the computation time, an "approxQuantile" function from the Apache Spark library can be used. In order to get 10 similarly sized bins, deciles are used.

Table 5.1: Profit × Churn classes

		Churn bins		
		1	$\dots$	j
Profit bins	*1*	$segm_{1,1}$	$\dots$	$segm_{1,j}$
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	i	$segm_{i,1}$	$\dots$	$segm_{i,j}$

The table 5.1 shows how the Profit×Churn target segments are created. If deciles are used to discretize source variables($i = 10, j = 10$), 100 almost equal-size segments are created.

Classification Tree

A simple principle used in RFM models can not be used for microsegmentation. The RFM approach would lead to $i \times j \times k$ number of microsegments (Profit, Churn and Behavioral segment combinations). Having $10 \times 10 \times 5 = 500$ microsegments would not only lead to a significant complexity of the Markov Chain model and Markov Decision process, but as well the impact of marketing incentives could be hardly measured.

Instead, an assumption that target Profit×Churn segments are affected by the customer's behavior is applied. Due to this fact, the task can be transformed into a machine-learning classification task, which takes the behavioral patterns as predictors and Profit×Churn classes as a target variable.

Predictors to be used in the classification are: previously created behavioral segments, (Ekinci et al., 2012) further suggest e.g. product/service types that customer used so far, customer loyalty and satisfaction level, demographic information, and operational risk.

The algorithm searches for similar patterns among the predictors while trying to distinguish the target (Profit×Churn) classes. The predicted class (model output) is then considered to be a single microsegment.

An ideal algorithm for this kind of tasks is the decision tree[3]. Firstly, for the possibility of result implementation in RDBMs via set of conditions and secondly tree-pruning allows to specify the minimum number of instances in a leaf (i.e. the minimum size of a microsegment).

The distributed decision tree algorithm is implemented in Spark Machine Learning libraries (spark.mllib.tree.DecisionTree). It allows to specify the *minInstancesPerNode* parameter, i.e. the minimum size of a microsegment. Based on the author's experience, having millions of clients, the parameter is set to $\approx 0.025\%$ of a customer base size.

5.1.2 Transition Function, Reward Vector

Once the microsegments (states) are defined, they can be back-propagated to the previous time periods and the transition probabilities observed as follows:

$$p(s'|s) = \frac{\sum_{t \in T} \frac{n_t^{s'|s}}{n_t^s}}{T} \tag{5.3}$$

Where

- T = number of time periods
- $n_t^{s'|s}$ = number of customers migrating from state s to s' in period t
- n_t^s = number of customers being in state s in period t

An average transition probability is used in the equation 5.3 in order to ensure the model stability, i.e. to mitigate the risk of exceptional behavior of small microsegments.

A similar approach is applied in the reward calculation[4]. A state reward is defined as an estimated profit within one period, as shown in equation 5.4.

$$r(s) = \frac{\sum_{t \in T} \bar{\pi}_t^s}{T} \tag{5.4}$$

[3]More information about the algorithm and its implementation can be found at: https://spark.apache.org/docs/latest/ml-classification-regression.html

[4]A constant reward is sufficient as the variance of profit per microsegment over time is low. Note that this assumption may be affected on developing markets or in countries with high inflation rate.

Where $\bar{\pi}_t^s$ = average profit per customer in state s in period t

5.1.3 CLV estimation

As all of the Markov Chain model components $\langle S, P, R \rangle$ were defined, CLV can be estimated regarding to the equation 4.3. The value of a whole customer base is then calculated as a sum of microsegment CLVs multiplied by the actual number of customers in each microsegment as proposed in the equation 4.4.

Python implementation of the CLV calculation algorithm is listed in the appendix A.

A simulation of the Markov Chain model is visualized in the figure 5.3. The figure reveals the flows (defined by components of the transition function $\mathbf{P}(\mathbf{s}'|\mathbf{s})$) between customer microsegments throughout the time periods (t_0, t_1, t_2 and t_3).

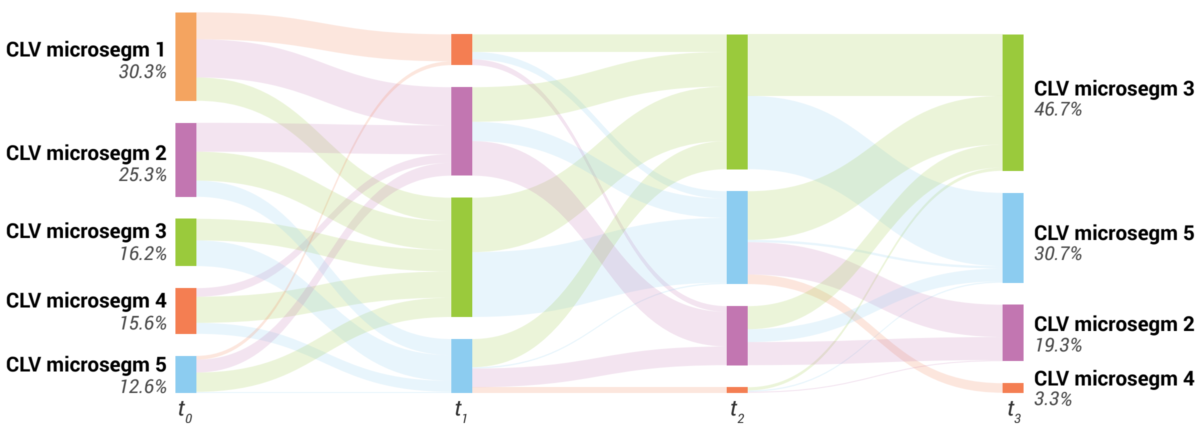

Figure 5.3: Example of a Markov Chain model represented by the Sankey diagram, Source: Author

The diagram (5.3) is particularly valuable from the business perspective as it displays the customer dynamics in more understandable way compared to the transition matrix. It allows non-technical users to estimate the proportions of customer microsegments in the future throughout the whole customer portfolio.

5.2 CLV optimization throughout marketing campaigns

The previously developed Markov Chain model for CLV calculation is a starting point for the Markov Decision Process algorithm as it provides necessary CLV values of all microsegments, additionally the MC transition function represents a customer's "natural behavior". The natural behavior is defined as common behavior of a customer not targeted by any of the marketing incentives. However, firstly it is problematic to ensure that the customer was not reached by any of the launched marketing campaigns (e.g. television advertisements, outdoor billboards, or internet banners), secondly the reason of not-addressing the particular customer must be determined. If the customer was intentionally not targeted (e.g. being in a marketing control group), then the definition is fulfilled. On the other hand, in highly competitive industries, companies cannot afford to separate an important part of the customer's portfolio only for benchmark reasons and consequently the non-targeted customers can be described as those, who were not suitable for any of the past marketing campaigns. Those customers can be regarded as outliers as the marketing campaigns are designed to fit ordinary customers. Therefore, such group of clients cannot be referred as a representative sample and does not fit the definition. As the decomposition of marketing impacts on the customer behavior is unobtainable, non-adjusted Markov Chain transition probabilities are taken as natural customer's behavior.

5.2.1 Actions

Every marketing campaign is represented by at least one action in the MDP model, additionally the "no-campaign" action a_0 is defined. In cases when costs per campaign depend on the customer microsegment, multiple instances of the same marketing campaign with different reward functions may occur in the MDP model. In the modeled MDP an average cost per campaign is assumed and therefore $n+1$ actions exist, where n stands for the number of marketing campaigns available. No-campaign action (a_0) is related to zero costs and the transition probabilities are taken from the MC model, as previously discussed.

5.2.2 Transition Functions

Action specific transition probabilities are obtained similarly to the values calculated in the MC model. The only change is that the campaign result might not be observable in the period following the campaign execution, and consequently a time windows w has to be added to the computation, as stated in 5.5.

$$p(s'|s,a) = \frac{\sum_{t \in T-w} \frac{n_{t+w}^{s'|s,a}}{n_t^{s,a}}}{T-w} \tag{5.5}$$

Where

- $n_{t+w}^{s'|s,a}$ = number of customers in segment s' in period $t+w$ targeted by action a
- $n_t^{s,a}$ = number of customers in segment s in period t targeted by campaign a
- T = number of periods
- w = time window length

The time window w variable enables to measure the impact of marketing campaigns related to products or services that cannot be immediately processed (e.g. mortgages). The time window is also needed to incorporate the customer's decision time. On the other hand, what longer the time window is, the less precise transition probability is obtained as it is affected by the natural clients behavior. Therefore, the recommended time window is 3 months.

5.2.3 Reward Matrices

Action specific rewards based on the microsegment CLV values are defined as follows:

$$r(s'|s,a) = p(s'|s,a) \times \Delta CLV(s'|s) - c(a) \tag{5.6}$$

Where

- $p(s'|s, a)$ = transition probability
- $\Delta CLV(s'|s)$ = difference of CLV values observed from the MC model
- $c(a)$ = cost per action

In order to proceed to MDP modeling, calculated rewards $r(s'|s, a)$ based on the equation 5.6 need to be transformed into rewards depending only on the source state and the action taken $r(s|a)$. The previously mentioned equation 4.6 is used for the aggregation.

5.2.4 MDP construction

Since all the MDP components $\langle S, A, P, R \rangle$ were defined, the modeling can proceed to MDP construction. A MDPtoolbox library developed by (Chadès, Chapron, Cros, Garcia, & Sabbadin, 2014) provides a multiplatform (MATLAB, GNU Octave, Scilab, R and Python) implementation of algorithms to solve a wide range of MDPs. The library includes algorithms capable of both Dynamic Programming (DP) and Reinforcement Learning (RL). Moreover, it covers all the algorithms discussed in chapter 4.2.2.

Suggested technologies for the MDP implementation are Python and R – according to ("Spark Documentation", 2018) both programming languages include direct connectors to Apache Spark (used previously for data preparation), and MDPtoolbox libraries are available in related repositories.

Unconstrained MDP & N-best marketing campaigns per microsegment

The optimization is formulated as searching for the policy π that maximizes the value function V. The optimal policy π^* then defines the most appropriate action for each customer state. This strategy does not reflect any business or capacity constraints, and therefore the usage is very limited.

To handle customer eligibilities and previously mentioned business and capacity constraints, the MDP model must be modified. In general, the transition function $\mathbf{P}$ components define the probabilities. To avoid selecting previously excluded action a in state s, the linked transition probabilities $p(s'|s,a)$ must be set to zero. However, a change in the transition function further results in rewards $\mathbf{R}$ re-computation. Instead, the associated component of the reward matrix $r(s,a)$ can be directly set to a highly negative value and consequently the MDP algorithm avoids selecting this action. By iteratively adjusting the reward matrices, n-best actions can be obtained, where n stands for number of iterations.

Python MDP implementation

A Python version of the MDPtoolbox library[5] *pymdptoolbox* is used to find the optimal policy π^* for the given MDP. The algorithm further iterates as described in the previous section to obtain n-best actions by altering rewards $\mathbf{R}$.

Python implementation of the optimization algorithm proposing n-best marketing campaigns per customer microsegment (state) is listed in the appendix B.

Results validation

From the business perspective, an interpretability of MDP results is much more understandable than in case of deep learning algorithms (e.g. Artificial Neural Networks), or in ordinary models such as logistic regression where the exponential transformation might be hard to imagine for some of the users. On the contrary, MDP results of the favored marketing campaign on the desired customer microsegment can be visualized, as presented in the figure 5.4.

[5]MDP Toolbox library for Python installation steps and documentation can be found at: http://pymdptoolbox.readthedocs.io/en/latest/index.html

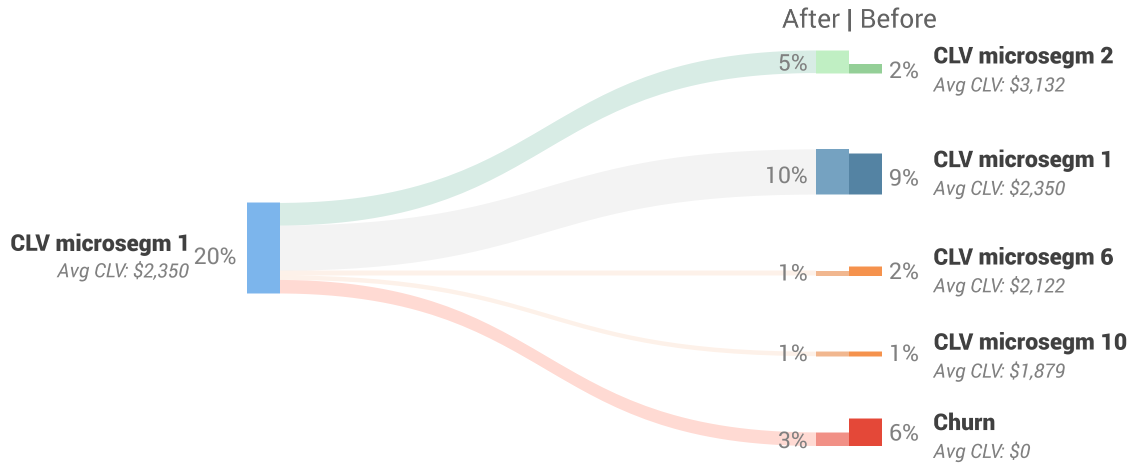

Figure 5.4: The impact of a preferred marketing campaign targeted on the selected customer microsegment, compared to the natural customer behavior, Source: Author

The figure 5.4 displays only data already available in the MDP model, therefore no additional computation is needed. The CLVs displayed are pre-computed in the MC model, transition probabilities are extracted from the MDP transition function. The diagram provides users with valuable insights of estimated client behavior and the financial impact through CLV. Moreover, the chart can be used as a powerful tool for model validation.

Summary

Firstly, CLV estimation method based on the Markov Chain model is introduced. Customer microsegmentation reflecting behavioral patterns, customer profitability and churn score must be created, which is considered to be the most challenging task (Cheng et al., 2012). Customer transition probabilities regarding previously defined customer states (microsegments) are obtained and the state rewards computed. Once all the MC model components are defined, the CLV value per microsegment is calculated.

Secondly, the MDP model is created by extending the MC model by actions (marketing incentives). MDP algorithms *Policy Iteration, Value Iteration and Q-Learning* are applied to retrieve the optimal policy π^* which defines the most suitable action per customer state (microsegment). To obtain n-best marketing campaigns per customer microsegment, the optimization is run iteratively.

Finally, possible visualizations of both models are mentioned for better model interpretability.

Chapter 6

Model Validation & Results

The aim of this chapter is to summarize findings gained throughout the MDP model implementation in a retail banking environment. Due to confidentiality reasons, adjusted, approximated and anonymized results are provided.

6.1 Data Description

The dataset describes customer behavior of European bank's clients between years 2012 and 2014. It contains data from various banking systems e.g. CRM, product history, transactional data, and account balances.

The key dataset drawbacks can be identified as:

1. No data about campaign costs are provided, therefore the Gamma distribution of campaign costs is assumed, while limiting the total amount spent on marketing expenditures at 10% of bank year revenue, as suggested by (Mullineaux & Pyles, 2010).

2. Only an aggregated profit quarterly is available in the dataset. As decomposition of the aggregated value and estimation of the profitability at the product/customer level would be very problematic and possibly lead to high discrepancies, revenue is used in CLV modeling instead of profit.

3. The data are given in a form of monthly snapshots, therefore the shortest time period in the modeling exercise is one month. Consequently, the impact of period length on CLV value cannot be examined.

The given banking dataset contains information about more than 5 million unique customers, their distribution over business segments is shown in the figure 6.1.

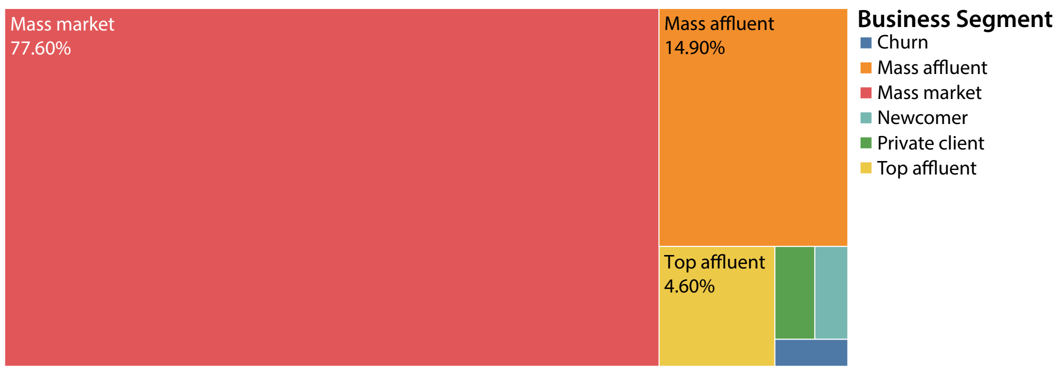

Figure 6.1: Business Segment distribution over customer base, Source: Author

Most of the customers belong to Mass market segment (77.6%), the second most frequent group is Mass affluent (14.9%), representations of each of the remaining business segments do not exceed 5%.

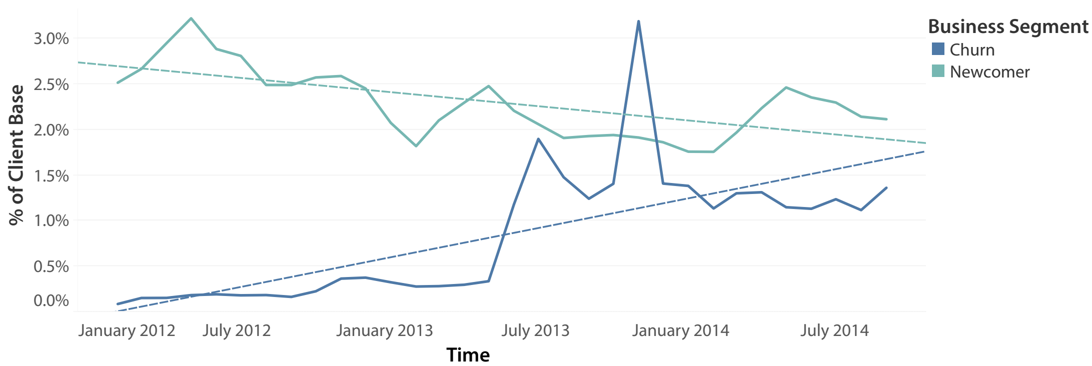

Figure 6.2: New & Churned customer percentages over time, Source: Author

The figure 6.2 displays an unsatisfactory trend of new and churned clients. While counts of newcomers are decreasing, the percentages of churned customers are accelerating. Simple linear regression estimations (represented by the dashed lines) show that in short time period the number of leaving customers can exceed the number of acquired ones. As a result, the client base would start diminishing.

Resolution of such problem may require transformation of business processes or product portfolio. However, a significant enhancement can be reached via optimization of marketing campaigns while focusing on the long-term oriented metric – Customer Lifetime Value. This approach is further developed in this chapter.

6.2 CLV modeling

The CLV modeling was performed as proposed in chapters 4 & 5. The optimal number of behavioral clusters based on WSSE was set to 5 (according to the figure 5.2). Due to lack of profit related measures, revenue was used instead. The revenue histogram and suggested discretization ranges are shown in the figure 6.3.

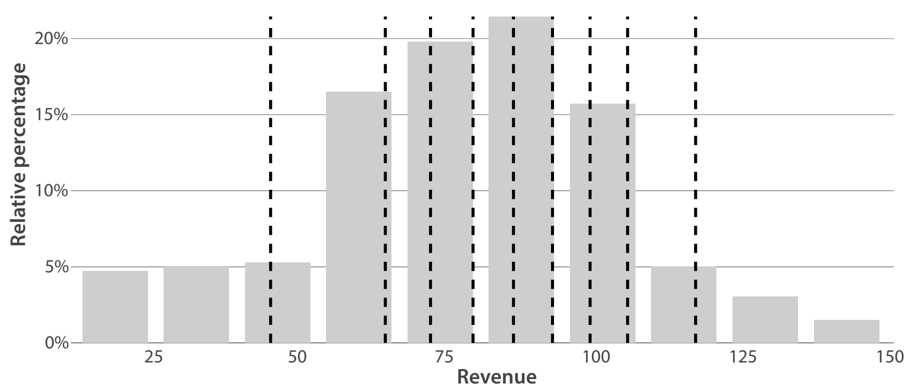

Figure 6.3: The revenue distribution with salient deciles, Source: Author

Decile discretization was used to create Revenue and Churn score bins, consequently 100 target classes were created. The classification algorithm (decision tree) was trained on behavioral data with the stopping criterion of a node size set at 25,000 customers. The algorithm created 116 mutually exclusive microsegments. Transition function and customer state rewards (average microsegment revenue) were calculated to prepare all the MC components.

Because of bank internal regulations and asset evaluation policies - finite time horizon of 2 years (24 periods) with the annual discount rate 2% were taken as input parameters into the CLV computation. Twelve different CLV models based on preceding 12 months data were built to validate the model stability. A sample of 3 microsegments and their estimated CLV value development can be seen in the figure 6.4.

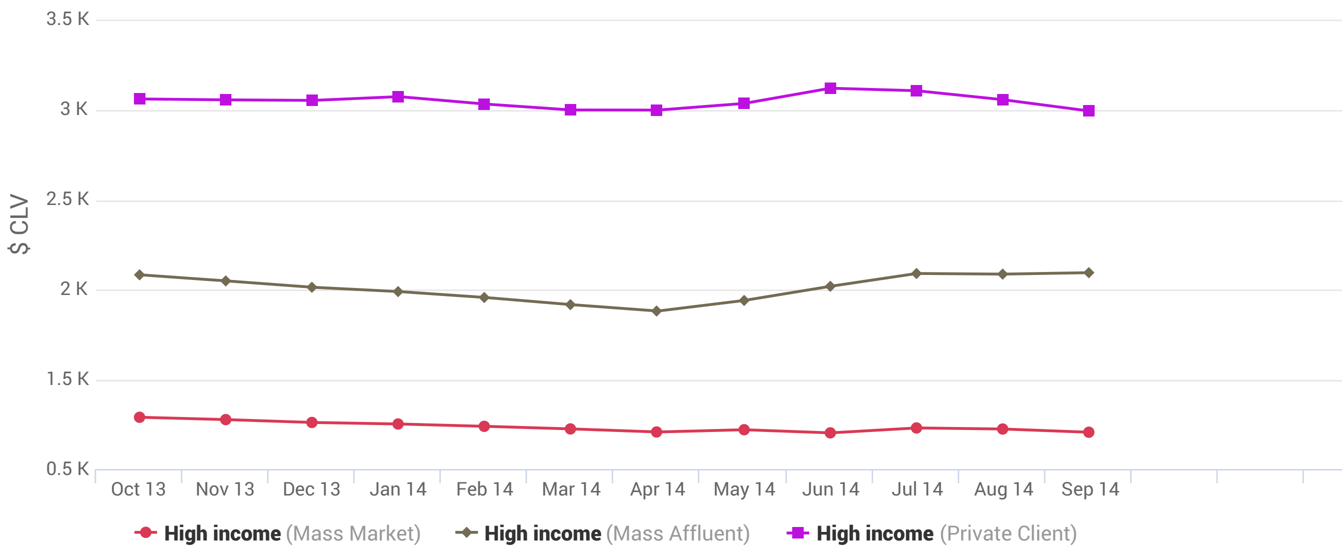

Figure 6.4: Estimated CLV development on selected microsegments, Source: Author

Since the deviation of estimated CLV value was sufficiently low, the MC model was regarded as stable. Therefore, the model outputs could be further used in CLV optimization.

6.3 CLV optimization

The previously defined customer microsegments, their CLVs and the "no-action" transition probabilities were used in the MDP construction, additionally marketing campaign specific transition probabilities and rewards had to calculated.

Three different MDP algorithms - *Policy Iteration, Value Iteration and Q-Learning*, were used to model the optimal policy π^*. As expected, the performance of dynamic programming algorithms (Policy Iteration and Value Iteration) outperformed the algorithm based on reinforcement learning (Q-Learning). Not only by comparing the achieved value of *Value function*, but also the algorithm runtime, which was in case of Q-Learning significantly higher. Since the problem was modeled as a stationary MDP (non-changing transition probabilities and rewards), the exploration strategy of the Q-Learning did not worth it. *Policy Iteration* and *Value Iteration* algorithms ended up with almost identical results. However, the *Policy Iteration* algorithm resulted in a slightly higher values of *Value function* and therefore it was selected for marketing strategy estimation.

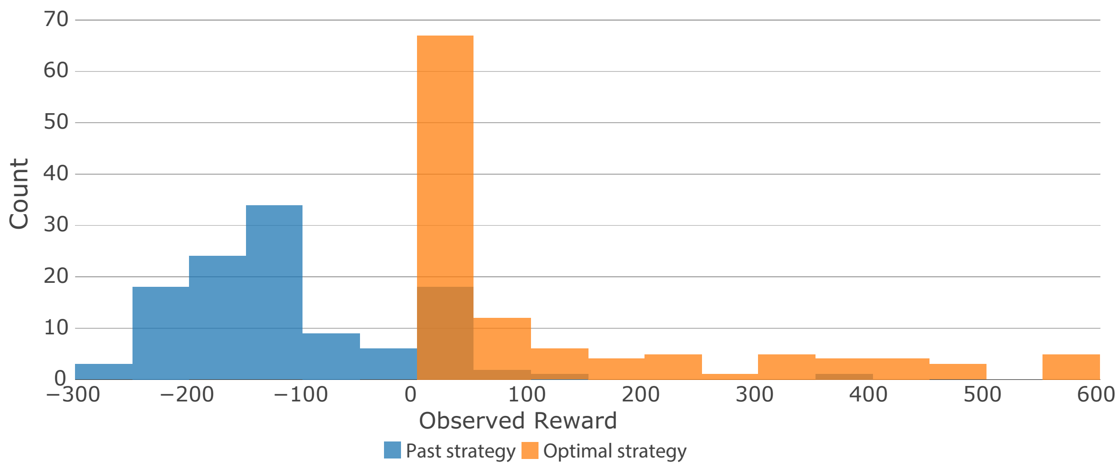

Figure 6.5: Comparison of optimal and past strategies, Source: Author

The figure 6.5 displays a comparison of marketing strategies performed by the bank in past and the optimal targeting strategy estimated by the MDP model. The optimal strategy does not reflect customer eligibility, budget or channel capacity constraints and therefore it cannot be referred as a feasible marketing strategy. Instead, it can be used for benchmark purposes and what-if analysis (e.g. what would be the impact on microsegment CLV, if the marketing budget was increased).

By removing the optimal marketing campaign from the MDP action space, the algorithm is forced to select a next-best marketing campaign. This behavior can be repeated until the action "do nothing" is reached. The result of such modified algorithm is an ordered set of n-best marketing campaigns optimizing CLV, which can be directly implemented in the campaign management system. The table 6.1 shows the output for 10 microsegments.

Table 6.1: MDP model output - estimated marketing strategy per microsegments

	Campaign order									
Microsegment	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
1	0	0	0	0	0	0	0	0	0	0
2	14	3	71	39	12	42	85	18	67	0
3	14	3	71	39	12	42	85	18	67	0
4	14	3	71	39	12	42	85	18	67	0
5	96	19	14	89	3	60	71	83	11	69
6	77	14	3	71	16	26	17	48	57	65
7	29	14	3	60	71	83	20	21	48	81
8	24	56	36	14	20	3	71	45	83	64
9	51	1	14	46	3	60	71	100	15	96
10	58	43	14	3	60	71	16	57	61	7

Columns in the table 6.1 show the n-optimal action. Value '0' stands for "do-nothing". Business implications can be derived from the result table, e.g. targeting Microsegm1 (churned clients) is not profitable. This behavior has 2 possible explanations: (1) the probability of a churned client acquisition is very low or (2) bank's retention processes are not well developed. Another interesting insight can be identified among microsegments 2, 3 and 4 – the identical marketing strategy can be explained by either lack of customized marketing campaigns for these microsegments, or high correlation among them and therefore possibility to merge and simplify the model.

Summary

The proposed modeling approach from chapters 4 & 5 was successfully verified on the retail banking dataset of more than 5 million customers. The customer base was split into 116 customer microsegments, for which the CLV was estimated. CLV optimization was performed by MDP model based on 101 actions (100 marketing actions plus the action representing natural customer behavior). The optimal strategy was then compared with the existing to exhibit the model benefits. The optimization task was run iteratively to obtain an ordered list of marketing campaigns suitable for each microsegment, which can be directly implemented in the campaign management system.

Chapter 7

Conclusion

In this thesis, I discussed allocation of marketing resources in order to increase the customer equity in retail banking. Despite traditional metrics used within the industry to optimize marketing campaigns such as profit or revenue, I introduced a customer centric model calculating Customer Lifetime Value (CLV) based on a Markov chain principle. Even though the CLV definition is in theory relatively simple (CLV is defined as the sum of all future discounted profits), two main obstacles are identified. Firstly, the calculated value is highly dependent not only on the modeling approach, but also on other company-specific factors e.g. discount rate or time horizon. Secondly, managerial goals are primarily related to short-term financial results, which are not necessarily associated with the CLV value. Whereas the first point can be solved by establishing a standardized CLV calculation methodology, to overcome the managerial issue, an entire transformation of the company approach is necessary. Regarding the literature review, the CLV based approach is considered to be the only sustainable strategy for future profitability and growth. Therefore, institutions should change the way how the performance is measured.

To ensure the consistency of solution components *(campaign management framework, CLV estimation and campaign optimization)* the comprehensive, industry tailored marketing campaign framework was developed. Although the framework is technologically independent, I strongly recommend using a single platform for the whole solution, which minimizes the need for data transmission and significantly improves the overall efficiency.

The fundamental part of the Markov Chain (MC) model used for CLV modeling is the state definition. The states are represented by customer microsegments (groups of customers with similar characteristics that are differentiable from each other) reflecting three main drivers of CLV: churn score, generated profit and behavioral pattern. The clustering is performed by a decision tree algorithm where the final nodes represent desired customer states. The CLV estimates were modeled based on transition probabilities (observed from the historical data) and average microsegment monthly revenues realized within the preceding year.

Since the CLV values and marketing campaigns were obtained, the last remaining step was to model the Markov Decision Process (MDP) optimizing the customer equity based on past marketing campaigns data. The model was run on the same customer states as the Markov Chain model, to ensure consistency of results. The Policy Iteration showed the best performance among multiple tested MDP algorithms.

The MDP model found the optimal action for each microsegment with respect to CLV, moreover the approach was successfully verified on the retail banking dataset of more than 5 million customers. For this reason, I consider the aim of the thesis to be entirely fulfilled.

In reality business, channel capacity, or customer eligibility constraints sometimes do not allow the targeting of each customer with their most optimal offer. Instead, by iteratively removing the optimal marketing strategy and rerunning the MDP algorithm, the ordered list of suitable marketing campaigns for each microsegment was created. The created list of campaigns can be directly used in the campaign management system to align the marketing strategy with the CLV objectives.

The source data had to be adjusted for confidential reasons, therefore the model outputs are mentioned not to prove the model performance but rather to illustrate the applied method and possible outcomes.

Future research

In my opinion the stationarity of both the MC and MDP models can be regarded as the most limiting factor. In the region of Central Europe, the proposed model results were stable as was discussed in the previous chapter. However in developing regions, the dramatic macroeconomic changes could lead to significant error terms.

Even though the data operations were processed in a distributed way, the MC and MDP algorithms run on a single machine. Especially when considering non-stationary Markov models, the reinforcement learning algorithms would be significantly slowing down the overall solution. If possible, these algorithms should be re-implemented to take the advantage of distributed computing.

Acronyms

ANN Artificial Neural Network.

CE Customer Equity.

CLV Customer Lifetime Value.

CP Customer Profitability.

CRM Customer Relationship Management.

DP Dynamic Programming.

GDPR General Data Protection Regulation.

LSE Least Square Error.

MC Markov Chain.

MDP Markov Decision Process.

MLE Maximum Likelihood Estimation.

RDBM Relational Database Management System.

RFM Recency-frequency-monetary.

RL Reinforcement Learning.

ROI Return on Investment.

SCV Single Customer Value.

SME Small and Medium-sized enterprise.

SOX Sarbanes-Oxley Act.

VAR Vector Autoregressive.

WSSE Within Set Sum of Squared Errors.

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Appendix

A Markov Chain CLV model

```
import numpy as np
import pandas as pd
from copy import copy

# Transition matrix
P = [[0.80, 0.05, 0.05, 0.10],
     [0.01, 0.95, 0.02, 0.02],
     [0.01, 0.04, 0.90, 0.05],
     [0.03, 0.03, 0.03, 0.91]]

R = [300,500,800,1000] # Rewards

ir = 2 # Discount rate 2% annualy

length = 72 # 6 years ahead

def calc_CLV(P, R, ir, period_cnt):
    """
    Calculate Customer Lifetime Value based on given parameters
    assume the length of one period = one month
    :param P: state transition matrix
    :param R: vector of rewards
    :param ir: interest rate (yearly)
    :param period_cnt: time horizon for CLV calculation
    :return: vector of CLVs per microsegment
    """
    discount_rate = (1+ir/100.0)**(1/12.0)-1
    CLV = copy(R)
    for period in range(period_cnt):
        state = np.dot(
                        np.eye(len(R)),
                        np.linalg.matrix_power(P, period+1)
                      )
        reward = np.dot(state, R)
        discounted_reward = reward/((1+discount_rate)**(period+1))
```

```
        CLV = np.add(CLV, discounted_reward)
    return np.append(period_cnt, np.round(CLV, 2))

clvs = {}
for i in range(length+1):
    clvs[i] = calc_CLV(P, R, ir, i)

# Transform the dictionary into a dataframe
CLVs = pd.DataFrame(clvs).transpose()

# Rename columns
CLVs.columns = np.append('Period',
                         ['Microsegm'+str(i+1) for i in range(len(R))])

# Save the output data
CLVs.to_csv('data/MC.csv', sep=';', index=False)
```

B Markov Decision Process Simulation

```python
# libraries
import pandas as pd
import numpy as np
import mdptoolbox as mx
from collections import OrderedDict
from copy import copy

# ==========================================================
# Parameter definition
# ==========================================================

# CLV params
# ----------------------------------------------------------
state_cnt = 30 # number of possible states
state_clv_mu, state_clv_sigma = 100, 50 # CLV distribution parameters

# Transition matrix params
# ----------------------------------------------------------
p_mu, p_sigma = 0, 2 # Transition probabilities distribution parameters
null_value = 10**(-10) # Value close to 0 that is used instead of null
                       # (for Markov Chain modeling)

# Actions - costs and total count
# ----------------------------------------------------------
C_min, C_max = 0.05, 20
action_cnt = 5 * state_cnt # number of campaings (actions) excluding
                           # "no-campaign" = natural customer's flow

# MDP
# ----------------------------------------------------------
discount =(1+0.02)**(1/12.0)-1 # 2% annualy

# Number of marketing action scenarios
# ----------------------------------------------------------
scenario_cnt = 10

# ==============================
# Generate state CLVs
# ==============================
np.random.seed(1)
state_CLV = np.abs(np.random.normal(loc=state_clv_mu,
                                    scale=state_clv_sigma,
                                    size=state_cnt))

# ==================================
# P = Transition matrix of actions
# ==================================
def getTranMatrix(p_mu, p_sigma, null_value,
                  state_cnt, action_cnt, seed = -1):
    if (seed != -1):
        np.random.seed(seed)
```

```
    t = np.abs(np.random.normal(p_mu, p_sigma, state_cnt*state_cnt))
    for i in range(len(t)):
        t[i] = null_value if t[i] > p_sigma else t[i]
    t = t.reshape(state_cnt, state_cnt)
    return t/t.sum(axis=1, keepdims=True) # normal. rows (row_sum = 1)

P = []
for i in range(action_cnt+1):
    P.append(getTranMatrix(p_mu=p_mu, p_sigma=p_sigma,
                           null_value=null_value, state_cnt=state_cnt,
                           action_cnt=action_cnt, seed = i))

# ================================
# C = Action costs
# ================================
def getActionCost(C_min, C_max, seed=-1):
    if (seed != -1):
        np.random.seed(seed)
    return -((C_max - C_min)*np.random.rand()+C_min)

C = []
for i in range(action_cnt+1):
    C.append(getActionCost(C_min=C_min, C_max=C_max, seed=i))
C[0] = 0 # no action (natural client's behavior)

# ================================
# R = Rewards
# ================================
def getRewards(P, state_CLV, C):
    rewards = []
    def deltaState(state_CLV):
        deltas = []
        n = len(state_CLV)
        for s0 in state_CLV:
            for s1 in state_CLV:
                deltas.append(s1-s0)
        return np.reshape(deltas, (n, n))
    delta = deltaState(state_CLV)
    for i in range(len(P)):
        rewards.append(np.multiply(P[i], delta)
                       .sum(axis=1, keepdims=True) + C[i])
    return np.transpose(rewards)[0]

R = getRewards(P=P, state_CLV=state_CLV, C=C)

# ********************************
# RESULTS
# ********************************
results = OrderedDict()
results['PolicyIteration'] = mx.mdp.PolicyIteration(
    transitions=P, reward=R, discount=discount)
results['PolicyIterationModified'] = mx.mdp.PolicyIterationModified(
        transitions=P, reward=R, discount=discount)
results['QLearning_10^4'] = mx.mdp.QLearning(
    transitions=P, reward=R, discount=discount)
```

```
results['QLearning_10^5'] = mx.mdp.QLearning(
    transitions=P, reward=R, discount=discount, n_iter=10**5)
results['ValueIteration'] = mx.mdp.ValueIteration(
    transitions=P, reward=R, discount=discount)
results['ValueIterationGS'] = mx.mdp.ValueIterationGS(
    transitions=P, reward=R, discount=discount)

for key, result in results.items():
    result.run()

for algorithm, result in results.items():
    print algorithm + ':'
    print '------------------------------'
    print np.round(result.time,3), sum(result.V)
    print np.array(result.policy).astype(int)
    print '\n'

# ********************************
# STRATEGY
# ********************************
def get_strategy(P, R, discount, scenario_cnt):
    reward = copy(R)
    strategy = []
    no_action = len(R)*[False]
    for i in range(scenario_cnt):
        result = mx.mdp.PolicyIteration(transitions=P, reward=reward,
                                        discount=discount,
                                        skip_check=True)
        no_action = no_action | (result.policy==0)
        result.policy[(no_action)] = 0
        strategy.append(result.policy)
        for j in range(len(result.policy)):
            reward[j][result.policy[j]] = -10**6
    return strategy

strategy = pd.DataFrame(np.transpose(get_strategy(P, R, discount,
                                                  scenario_cnt)))
strategy.columns = [i+1 for i in range(scenario_cnt)]
strategy['Microsegm'] = pd.Series(
  ['Microsegm'+ str(microsegment+1)
    for microsegment in range(len(strategy))])
strategy = strategy.set_index('Microsegm')
strategy.head(10)
```