Using Transformation in Solving Problems with Supplementary Information (Extended Abstract)

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The main task of this thesis is to examine the use of transformation in solving problems with supplementary information. The thesis is a continuation of the research devoted to various aspects of information (e. g. [Gaži, 2006]). It belongs to the part, where the topic of the research is the ”usefulness” of the information, while as a tool used for the measurement of the usefulness is the descriptional complexity of deterministic finite automata. The results presented to this date relied on the fact, that the supplementary information is in the form, which can be used directly.

In this thesis, we propose a framework for studying the possibility to transform the instance of a problem to match the format of the advisory information. A widely used and quite general transformation device is an a-transducer. It turned out, that it is not always convenient to allow the use of nondeterminism in transformations. Therefore, we examine also the case, that the transformation device is a sequential transducer. Both of these devices are defined and reviewed, e. g., in [Ginsburg, 1975].

The thesis is devoted to the notion of information and to the question, how can its various aspects be defined, studied and measured. The proposed thesis develops a model for studying the usefulness of information also in cases, where it is not usable directly. This model allows to consider the earlier results about the usefulness of information as a special case and therefore allows a direct comparison with the previous model.

Since we deal with the state complexity of finite state devices, we present an auxiliary result concerning the minimal state complexity of an one-bounded a-transducers, that realizes the a-transduction of ”modular-counting” languages, i. e. the languages of the form

$$L_k = \{a^k | k \equiv 0 \pmod{k}\}.$$

The tight lower bounds are shown by the following theorem.

Theorem 1. *For a pair of languages L_k, L_l, the minimal state complexity of an a-transudcer M, such that $M(L_k) = L_l$, is*

$$\min(l, \tfrac{\max(k,l)}{\gcd(k,l)}).$$

Furthermore, we study the classes of languages concering the possibility to simplify their complexity using the supplementary information. We define the class of the languages decomposable without the use of transformation ($\mathscr{D}_A$) and the class of the languages decomposable with use of the transformation by a sequential transducer ($\mathscr{D}_T$). Moreover, we present two possible definitions of the similar classes using the transformation by a-transducers ($\mathscr{D}_{NT_\forall}$ and $\mathscr{D}_{NT_\exists}$). The main result of this paper is the following comparison of these classes.

Theorem 2. $$\mathscr{D}_A \subsetneq \mathscr{D}_T \begin{array}{c} \subsetneq \mathscr{D}_{NT_\forall} \subsetneq \\ \\ \subsetneq \mathscr{D}_{NT_\exists} \subsetneq \end{array} \mathscr{R},$$

where $\mathscr{R}$ represents the class of regular languages.

References

[Gaži, 2006] Gaži, P. (2006). Parallel decompositions of finite automata. Master's thesis, Faculty of Mathematics, Physics and Informatics, Comenius University, Bratislava.

[Ginsburg, 1975] Ginsburg, S. (1975). *Algebraic and Automata-Theoretic Properties of Formal Languages*. Elsevier Science Inc., New York, NY, USA.

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