Algorithmic Analysis of Code-Breaking Games

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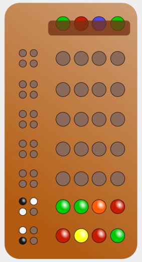
Code-Breaking Games

- 2 players: *codemaker* and *codebreaker*
- Codemaker selects a *secret code*
- Codebreaker strives to reveal the code through a series of *experiments* whose outcomes give partial information about the code
- Example: Mastermind
  - Secret code: combination of *n coloured pegs*
  - Codebreaker makes guesses (experiments)
  - Guesses are evaluated with *black and white markers*
  - Black marker = correct both colour and position
  - White marker = the colour is present at a different position



- Example: Counterfeit Coin
  - Problem of finding an odd-weight coin using balance scale
  - Secret code: identity of the unique counterfeit coin
  - Codebreaker puts coins on the balance scale and observes the outcome

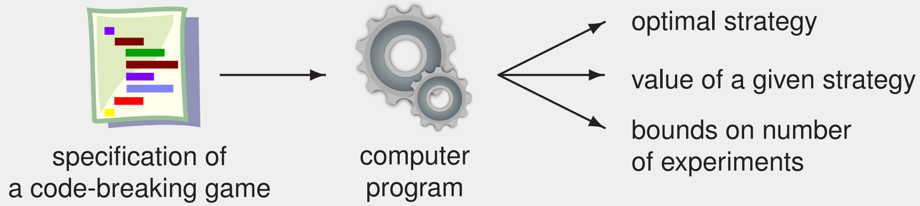
Questions and Problems

How should the codebreaker play in order to minimize the number of experiments needed to undoubtedly determine the code?

Is there a strategy for experiment selection that guarantees revealing the code after at most k experiments?

What strategy is optimal with respect to the average-case number of experiments, given that the code is selected from the given set with uniform distribution?

- **We created a computer program to automatically answer these questions**



Our Steps Towards Automatic Analysis

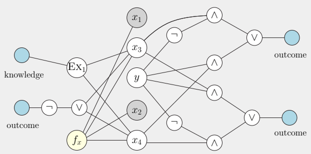
1. Creating a general, formal model of code-breaking games
   - Model based on propositional logic
   - Secret code = valuation of variables
   - Partial information = logical formula

$$\Phi_t = \{(f_x(\$1) \wedge \neg y) \vee (f_x(\$2) \wedge y),\ (f_x(\$1) \wedge y) \vee (f_x(\$2) \wedge \neg y),\ \neg f_x(\$1) \wedge \neg f_x(\$2)\ \}.$$

2. Proposing general strategies for experiment selection
   - *"Select an experiment that minimizes the maximal number of possibilities for the code in the next round"*
   - Several strategies of this kind formalized within the model

$$f(\Psi) = \frac{\sum_{\varphi \in \Psi} (\#\varphi)^2}{\sum_{\varphi \in \Psi} \#\varphi}$$

3. Developing algorithms for strategy evaluation and synthesis
   - Based on intelligent backtracking
   - Symmetry detection reduces the size of the state-space



4. Designing a computer language for game specification
   - Corresponds to the formal model
   - Built on top of Python for easier generation

```
for m in range(1, N//2 + 1):
  EXPERIMENT("weighing" + str(m), 2*m)
  OUTCOME("lighter", "((%s) & !y) ...
  OUTCOME("heavier", "((%s) & y)  ...
```

5. Implementing proposed algorithms in a computer program
   - Command-line tool written in C++
   - Use of modern SAT solvers for satisfiability queries needed by the algorithms

6. Using the program to create new and reproduce existing results
   - Easy reproduction of some of the existing results for Mastermind
   - Automatic analysis of generalizations and other code-breaking games